STRINGS TO REALITY: EFFECTIVE SUPERGRAVITY FOR COSMOLOGY AND PARTICLE PHENOMENOLOGY

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Stabilized string compactifications give rise to effective four-dimensional supergravities at energies below the compactification scale, and in this thesis, we investigate how these theories may be constrained by physical observables from particle physics and cosmology. A number of physically well-motivated scenarios such as cosmic inflation, intermediate-scale supersymmetry breaking, and a particular mechanism for baryogenesis, all rely on the form of the non-renormalizable, Planck-mass suppressed operators in the Lagrangian, which can be computed only in a quantum theory of gravity. With this motivation, we investigate supersymmetry breaking in stabilized compactifications of type IIB string theory, and find that experimental constraints from flavor physics impose severe restrictions on certain interesting compactification scenarios. Moreover, we point out that the Affleck-Dine scenario is incompatible with some models of string inflation, e.g. brane inflation in warped throats, and we show that versions of Affleck-Dine baryogenesis with non-trivial field dynamics during observable inflation induce features in the spectrum of the cosmic microwave background radiation, which may be observable by upcoming satellite experiments, such as Planck. Furthermore, there is compelling cosmological evidence for a 'dark energy' currently dominating the energy density of the universe, but accommodating this fact by direct constructions of de Sitter string vacua has proved challenging. Here, we use random matrix theory to show that in super-

gravities with many scalar fields, an exponentially small fraction of the de Sitter critical points are metastable vacua. This result significantly affects the counting of de Sitter vacua in the string landscape.

BIOGRAPHICAL SKETCH

David Marsh was born in Stockholm, Sweden and grew up in a family of five in the northern town of Östersund. Literature, sports, hiking, chess, politics and music were among the interests that he intensively pursued during various stages of his youth, and once he graduated from the science program of the Palmcrantz high-school in Östersund, he furthered his musical education by enrolling at a so called "folkhögskola" outside Örebro, Sweden,where he primarily studied the violin. Feeling the alluring pull of science, he enrolled in the Physics program at Uppsala University in the fall of 2002 and immediately developed a passion for mathematics and physics. He concluded his studies in Uppsala with a short research project under the supervision of professor Antti Niemi, and earned his M.Sc. with the thesis *"The Grassmannian Sigma Model in* $SU(2)$ *Yang-Mills Theory"*, in the winter of 2007. During the fall of 2007 he enrolled at the Cornell Physics Ph.D. program, and his time in Ithaca proved to include both tragic and joyful family events: in November of 2007 his brother Mathias, to whom this thesis is dedicated, passed away in cancer, and in June of 2009, he married his long-time girlfriend Marit Strömberg. At Cornell University, his research in high-energy theory focused on cosmological and phenomenological consequences of string theory.

Till *Mathias*, min älskade bror.

Till *Marit*, med tacksamhet och kärlek.

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INTRODUCTION

1.1 Fundamental physics in a data driven era

At the dawn of September 10, 2008, the Large Hadron Collider commenced its operation by circulating bunches of protons through its cold, underground tunnel at the feet of the French and Swiss Alps. At the time, the project had been under way for more than twenty years, and, as it turned out, it was still a full year from producing its first collisions. Currently, the collider is pushing the frontier of particle physics by operating at unmatched energies and intensities. The LHC explores energy scales at which the physics is largely unknown, and at which new discoveries are anticipated. In December 2011, the experiments ATLAS and CMS jointly announced tentative signs of a Higgs boson at energies around 125 GeV [1]. While the findings are not yet conclusive, the accumulation of more data during 2012 is expected to either confirm the existence of a Higgs boson, or decisively exclude it at these energies. The discovery of a Higgs boson would confirm the Standard Model explanation of the generation of mass and the breaking of the electroweak symmetry down to electromagnetism.

However, as we will review in this introductory chapter, strong theoretical arguments can be made for the Higgs boson not being the only new physics result that may be discovered at the LHC. In particular, an additional symmetry of space-time in the form of so called *supersymmetry*, is theoretically well-motivated to appear at this energy scale, and the experiments at the LHC is currently investigating whether this symmetry is realized in nature close to the electroweak scale or not.

Meanwhile, observational cosmology has evolved into a science of remarkable accuracy and predictive power. Over the past fifteen year, studies of distant supernovae have provided strong evidence for the existence of a positive vacuum energy density, or 'dark energy', which currently is dominating the energy density of the universe. Satellite experiments exploring minute deviations in the spectrum of the cosmic micro-wave background radiation (CMB) have provided substantial evidence for a period of inflation, or something very similar to it, taking place in the early universe. Current and upcoming experiments, such as Planck, will further explore the polarization of the cosmic microwave radiation, and may within the coming year determine the energy scale of inflation. Experiments studying the large-scale structure of the universe, as well as dark matter, have also contributed to the progress in cosmology, and taken together, these experiments have prompted scientists to describe the current era of cosmological research as a 'golden age'.

Consequently, the current epoch of physics offers an exploration of our universe from multiple angles and energy scales, ranging from the small scales probed at the LHC to the large scales of the cosmos, and experimental *data* is guiding the way into these yet unknown realms of reality. In the light of this progress, it is interesting to explore the limits of how much can be learned from the outcomes of these experiments — in particular, one may wonder how much we can learn about the underlying fundamental theory of quantum gravity. This theory may first become important at as high energies as the Planck scale, and since,

$$\frac{M_{EW}}{M_{Pl}} \approx 10^{-16}\,, \tag{1.1}$$

one may fear that the underlying structure of fundamental physics completely decouples from the relatively low energy physics currently probed at the LHC, and that it therefore will be extremely difficult to bridge the gap between the scale of the experiments and the scale of the fundamental physics. While such an argument may ultimately not be unfounded, we will in this thesis demonstrate how some of the most promising scenarios for particle physics and cosmology from string theory can indeed be constrained by experiments. For instance, in Chapter 2, we show that, once supersymmetry breaking is taken into account, experimental results from flavor physics impose severe constraints on some of the most successful models of particle physics in string theory [2]. Furthermore, it is well known that cosmic inflation is sensitive to high-scale physics such as string theory, and we note that this is also true for certain mechanisms explaining the asymmetry between matter and antimatter in the universe. In Chapter 3, we explain how such a mechanism may leave an imprint in the cosmic microwave background radiation, and how evidence for the mechanism automatically would be evidence against certain models of inflation in string theory [3].

In Chapter 4, we study the typical structure of the low-energy effective descriptions arising from string theory, and we pay particular attention to the stability of critical points in these theories. We show that meta-stable vacua are exceedingly rare as compared to the number of unstable critical points, and we argue that this affects the counting of vacua in string theory. For typical critical points these considerations turn the 'landscape' of string vacua into a 'wasteland' [4]. This work has important consequences for how one may anticipate to solve the cosmological constant problem in these theories. We note that certain non-typical critical points are more stable, and we conclude that while there is a landscape of string vacua, it appears quite different from prior expectations.

In sum, the experiments of the current data-driven era are pushing the boundaries of what is known about the electroweak scale and about the early universe. The experimental constraints that follow come with highly interesting opportunities for improving our understanding of fundamental physics. In this thesis, we illustrate this point by providing three examples of where experimental results from particle colliders, CMB experiments, and supernovae observations lead to constraints on the underlying high-energy theory.

1.2 Effective Field Theory from String Theory

In the remainder of this Chapter, we will review the background material necessary for understanding the research results of Chapters 2 through 4. In particular, this involves the structure of the Wilsonian effective quantum field theories arising from string compactifications as low-energy theories, supergravity and moduli stabilization. We will furthermore motivate the statistical studies of the effective theories discussed in Chapter 4 by reviewing the current understanding of the 'landscape' of string vacua.

1.2.1 Wilson's effective field theory

For the development of physics, effective descriptions have been of singular importance. For instance, Newton's highly successful theory of mechanics constitute an effective theory of matter, which breaks down and must be replaced by a more fundamental theory in certain limits, such as at high velocities or for microscopic amounts of matter. Quantum field theories have been highly successful in describing particle physics, with the 'Standard Model' of particle physics describing the physical world up to at least the electro-weak scale. However, quantum field theories which do not include gravity are always effective theories, and should be replaced by a theory of quantum gravity for processes at energies in which the fundamental theory may be important. In fact, the standard model may very well be an effective field theory description of a more fundamental, non-gravitational, theory at energies just slightly above the electro-weak scale, and the framework for describing how an 'ultra-violet' theory gives rise to an 'infra-red', low-energy description is the topic of effective field theory, to which we now turn.

There are two distinct notions of effective field theory, Wilson's prescription which involves integrating out high-scale physics to obtain an effective low-energy description [5], and the 1-particle irreducible (1PI) effective theory, in which modes at all masses contribute to the effective action. Here we will review some elementary properties of the Wilsonian effective theory, and only allude to the 1PI effective action in passing. These topics are treated in full detail in numerous textbooks and review articles, see e.g. [6].

Wilson's effective field theory is concerned with theories which are valid below some cut-off energy Λ_0, for which we are interested in processes occurring at a low energy scale E, where $E \ll \Lambda_0$. We may derive an effective theory valid below Λ satisfying $E < \Lambda < \Lambda_0$ by integrating out high-frequency modes. Schematically, a weakly-coupled field ϕ can be split into a high-frequency and a low-frequency part as $\phi = \phi_H + \phi_L$, where the frequencies of ϕ_H exceed Λ, while the frequencies of ϕ_L are less than Λ. By integrating out the high-frequency modes, i.e. by performing the path integral over ϕ_H, the Wilsonian low-energy theory for ϕ_L can be written as,

$$\int \mathcal{D}\phi_L e^{iS_\Lambda(\phi_L)}\,, \tag{1.2}$$

where $e^{iS_\Lambda(\phi_L)} = \int \mathcal{D}\phi_H e^{iS(\phi_L,\phi_H)}$. The action $S_\Lambda(\phi)$ typically differs from the high-scale action by values of coupling constants — or even by containing new couplings not present in the high-energy theory. By lowering the cut-off of the theory by a small amount, $\Lambda = (1-\epsilon)\Lambda_0$, for some small $\epsilon > 0$, the evolution of a coupling constant g_i of some local operator O_i of dimension Δ_i, can be viewed as a renormalization group flow in the space of Lagrangians. At weak coupling and to linear order, the coupling constant g_i with dimension $4-\Delta_i$ then satisfies the differential equation,

$$\frac{dg_i}{d\Lambda} = (\Delta_i - 4)\frac{g_i}{\Lambda}\,, \tag{1.3}$$

which is solved by,

$$g_i(\Lambda) = c_i\,\Lambda_0^{4-\Delta_i}\left(\frac{\Lambda}{\Lambda_0}\right)^{\Delta_i-4}\,, \tag{1.4}$$

for some dimensionless constants c_i. Operators with dimensions $\Delta_i > 4$ become less important at low energies and are called *irrelevant*, while operators with $\Delta_i = 4$ and $\Delta_i < 4$ are called *marginal* and *relevant*, respectively. The flow of relevant operators is divergent, signaling that significant fine-tuning in the high-energy theory of the order of $(m\mu/\Lambda_0^2)^{4-\Delta}$ is necessary for a relevant operator to appear with a modest value, $m^{4-\Delta_i}$, at the low scale μ. Relevant operators are therefore said to typically not be natural in effective theories. This point is evident also in the 1PI-effective action, which can be illustrated in a toy-model consisting of a light real boson, ϕ, of mass m, and a heavier Dirac fermion, Ψ, with mass M, and with an action given by,

$$S = \int d^4x \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + i\bar{\Psi}\partial\!\!\!/\Psi - M\bar{\Psi}\Psi + y\phi\bar{\Psi}\Psi\right). \tag{1.5}$$

The 1PI effective action for processes at energies $\mu \ll M \lesssim \Lambda$ is obtained by methods described in e.g. [6]. The effective boson mass in the low-energy theory with cut-off Λ, in which the fermion has been integrated out, is given by,

$$m_{eff}^2 = m^2 + \frac{y^2}{16\pi^2}\left(c_1\Lambda^2 + c_2 M^2 + c_3 m^2 \ln\frac{\mu}{\Lambda} + O\left(\frac{M^4}{\Lambda^2}\right)\right) . \tag{1.6}$$

Here, the dependence of the effective mass on the cut-off Λ is an artifact of this particular regularization method, however, the dependence on the fermion mass M, is not, and appears in any regularization scheme. For the effective mass to be small with respect to Λ and M, the one-loop corrections on the right-hand side of equation (1.6) must cancel to a high accuracy. This indicates that relevant operators introduce a detailed sensitivity to high-scale physics at scale Λ which does not decouple at low energies. In the standard model, the mass of the Higgs boson is the only relevant operator, and its un-naturalness is known as the gauge hierarchy problem.

This sensitivity to high-scale physics is particularly troublesome for the dimension zero operator $O = 1$, the vacuum expectation value of which is interpreted as a cosmological constant in the low-energy theory. Contributions from high-scale physics naturally make the coupling constant of this operator of order M_{Pl}^4 (at which point quantum field theory without gravity breaks down), however the observed value is around 120 orders of magnitude smaller than this. We will return to the cosmological constant problem in §1.5.

In contrast, the flow of irrelevant operators converges to small coupling constants (as the energy scale is decreased) and thereby acts to erase detailed information of the high-scale physics. On the other hand, by 'reversing' the flow of the irrelevant operators, we note that the effective theory breaks down as these become of the order of the cut-off scale. The Fermi theory of weak interactions provides an illuminating example of this: the low-energy effective theory contains no relevant operators, and one of its dimension six operators mediate muon decay to an electron, a μ-neutrino and a electron-anti-neutrino. The coupling constant in the effective theory, G_F, has dimension -2, and is given by the matching condition, $G_F = g^2 \frac{\sqrt{2}}{8M_W^2}$, in terms of the coupling g of the high-energy Weinberg-Salam theory. The effective theory breaks down at energies comparable to M_W, at which point more degrees of freedom — in this case the $W^{\pm}$-bosons — need to be included to render the theory consistent beyond the low-energy cut-off. In Chapters 2 and 3 of this thesis, we investigate physical processes in which Planck-mass-suppressed non-renormalizable operators are important. Since these operators are suppressed by the scale at which gravity is expected to become important, they can only be computed in a high-energy theory that includes quantum gravity, such as string theory. Hence, questions of this type provide a particularly clear window towards high-scale physics.

At the quantum level, the dimensions Δ_i deviate from the classical scaling dimensions due to the effect of interactions. For marginal operators at weak coupling, the quantum contributions give rise to a logarithmic running with the energy scale. For example, gauge couplings are marginal at tree-level, and run like,

$$\Lambda \frac{dg(\Lambda)}{d\Lambda} = b\, g^2(\Lambda) \tag{1.7}$$

at the one-loop level.[1] The sign of the b determines whether or not the coupling is *marginally relevant* or *marginally irrelevant*, and is determined by gauge group factors and the charged matter content of the theory. For the weak $SU(2) \times U(1)$ interactions, the sign of b is positive and the coupling becomes irrelevant

[1]The value of the coupling constant at scale Λ, given its value at scale Λ_0 is $g(\Lambda) = \frac{g(\Lambda_0)}{1+bg(\Lambda_0)\ln(\Lambda_0/\Lambda)}$.

in the infra-red, while for QCD, the coupling famously grows in the infrared, eventually making the theory strongly coupled at the scale,

$$\Lambda_{QCD} = \Lambda_0 e^{1/bg(\Lambda_0)} \approx 200 \ \ \mathrm{MeV}\,, \tag{1.8}$$

and the low-energy theory below this energy scale is a theory of mesons which exhibits chiral symmetry breaking. This generation of a hierarchy between the ultraviolet scale and the low-energy scale due to the logarithmic running of the coupling constant is called *dimensional transmutation*. It can be shown that non-Abelian gauge couplings are the *only* marginally relevant operators, and thus are the only marginal operators supporting dimensional transmutation [7].

Symmetries can change the renormalization group flow and make relevant operators technically[2] natural, as is exemplified by fermion masses: the theory satisfies an enhanced, chiral symmetry as $M = 0$, which means that corrections to the fermion masses must be proportional to M, and receive no linear contribution in Λ. Vectors can be made natural if they are gauged, and scalar bosons can be naturally light by shift symmetry or supersymmetry. Let us now turn to the latter topic, which is not only well motivated by low-energy effective theory considerations, but also by the effective theories arising from string theory, as we will discuss in §1.2.3.

1.2.2 Supersymmetry and supergravity

This aim of this section is to set up the supersymmetric formalism used in Chapters 2, 3, and 4, and to review some of the aspects of supersymmetry of partic-

[2]Here, the term 'technical' serves to distinguish this type of naturalness, where a coefficient of a relevant operator is small due to some symmetry in the limit of vanishing coefficient, to true naturalness, which would require an explanation in the ultraviolet theory of the origin and breaking of this symmetry.

ular importance in this work. We will discuss both global supersymmetry, and supersymmetry in the presence of gravity, but our presentation of both topics is necessarily brief. For more thorough and detailed reviews, see e.g. [8, 9, 10].

Supersymmetry is a non-trivial extension of the space-time symmetries of the Poincaré group (the generators of which include translations, rotations, and Lorentz boosts). In fact, inspired by the 'no-go' theorem by Coleman and Mandula [11], which states that there is no non-trivial extension of the Poincaré group consistent with a non-trivial S-matrix for tensor generators, supersymmetry was discovered as a "counter-example" based on Grassmannian, spinor, generators, and later proven to be the unique non-trivial extension of the Poincaré group that does not over-constrain the corresponding S-matrix [12]. This is not only a mathematically beautiful result, but supersymmetric theories are also very powerful, as supersymmetry can render relevant operators technically natural.

For instance, for unbroken supersymmetry the value of the cosmological constant vanishes, i.e. $\langle V(\phi)\rangle = 0$, and even for supersymmetry which is broken at scale m, the expected scale of the cosmological constant is smaller by a factor of $\left(m/M_{Pl}\right)^4$, as compared to the non-supersymmetric case. Supersymmetry also shields scalar masses from detailed sensitivity to ultraviolet physics, which in particular means that the one-loop contributions proportional to M^2 and Λ^2 in the equation (1.6) do not appear, and bosonic masses only run logarithmically with the energy scale. Historically, supersymmetry has also been motivated by the support they give for gauge coupling unification at scales $E_{GUT} = 10^{15}$ GeV, and for providing natural candidate states for dark matter.

Current bounds from the LHC on superpartner masses are very interesting in this context, and the fact that supersymmetry has not yet been found could be a hint that the supersymmetric solution to the gauge hierarchy problem is not realized in nature. On the other hand, many natural models of supersymmetry are still not ruled out by experiments, and whether or not any of these models are realized in nature will hopefully be settled within a few years. With this introduction, let us begin by reviewing the notation and basic features of global supersymmetry in Minkowski space.

Global $\mathcal{N} = 1$ supersymmetry in four dimensions

In $\mathcal{N} = 1$ global supersymmetry, fields can be organized in either chiral multiplets, vector multiplets, or linear multiplets. Linear multiplets are dual to chiral multiplets, so it will suffice to discuss the chiral (matter) fields, and vector (gauge) fields. The superspace formalism, in which space-time is extended to also include the Grassman spinors θ_α and $\bar{\theta}_{\dot{\alpha}}$, i.e. $x^\mu \to (x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$, is convenient for writing general actions that are invariant under supersymmetry.

In superspace, the supercharges can be written as $Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$, and $\bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$, and satisfy the supersymmetry algebra,

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0\,, \tag{1.9}$$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2P_{\alpha\dot{\beta}}\,, \tag{1.10}$$

where $P_{\alpha\dot{\alpha}} = P_\mu\sigma^\mu_{\alpha\dot{\alpha}}$. Covariant derivatives on superspace can be defined as, $D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$, and $\overline{D}_{\dot{\alpha}} = +\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$, in terms of which a *chiral superfield* can be defined as a field $\Phi(x, \theta, \bar{\theta})$ on superspace which satisfies,

$$\overline{D}_{\dot{\alpha}}\Phi = 0\,. \tag{1.11}$$

In terms of component fields which only depends on the superspace coordinate $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$, condition (1.11) gives,

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)\,, \tag{1.12}$$

where ϕ is a scalar, ψ a Weyl spinor, and $F(y)$ is an auxiliary, non-physical degree of freedom. The action for a collection of chiral superfields, denoted Φ^i, is completely specified in terms of a real Kähler potential $K(\Phi^i, \bar{\Phi}^{\bar{i}})$, and a holomorphic superpotential $W = W(\Phi^i)$ as,

$$\begin{aligned}
\mathcal{L} &= \int d^4\theta\, K(\Phi^i, \bar{\Phi}^{\bar{i}}) + \Big(\int d^2\theta\, W(\Phi^i) + h.c.\Big) = & (1.13)\\
&= K_{i\bar{i}} F^i \bar{F}^{\bar{i}} - K_{i\bar{i}}\, \partial_\mu\phi^i\partial^\mu\phi^{*\bar{i}} - iK_{i\bar{j}}\, \bar{\psi}^{\bar{j}}\bar{\sigma}^\mu\Big(\partial_\mu\psi^i + \Gamma^i_{jk}\partial_\mu\phi^k\psi^j\Big) \\
&\quad -\frac{1}{4}K_{i\bar{j}k\bar{l}}\,\psi^i\bar{\psi}^{\bar{j}}\psi^k\bar{\psi}^{\bar{l}} - \frac{1}{2}\Big(F^i K_{i\bar{i}}\,\bar{\Gamma}^{\bar{i}}_{\bar{j}\bar{k}}\,\bar{\psi}^{\bar{j}}\bar{\psi}^{\bar{k}} + h.c.\Big) \\
&\quad +\Big(W_i F^i - \frac{1}{2}W_{ij}\psi^i\psi^j + h.c.\Big)\,. & (1.14)
\end{aligned}$$

Here, a sub-index on K and W denotes a partial derivative, and $K_{i\bar{i}}$ is the Kähler metric on field space, with inverse given by $K^{i\bar{i}}$. With this metric, the field space is a Kähler manifold and consequently the Christoffel symbols on field space, Γ^i_{jk} and $\overline{\Gamma}^{\bar{i}}_{\bar{j}\bar{k}}$, have no non-vanishing components with mixed holomorphic and anti-holomorphic indices. Consistently, the action is invariant under Kähler transformations: $K(\Phi,\bar{\Phi}) \to K(\Phi,\bar{\Phi}) + f(\Phi) + f^*(\bar{\Phi})$.

From equation (1.14), the algebraic equation of motion for the auxiliary field F^i is solved by $F^i = \frac{1}{2}\Gamma^i_{jk}\psi^j\psi^k - K^{i\bar{i}}\overline{W}_{\bar{i}}$, which upon substitution back into the Lagrangian gives the scalar potential,

$$V(\phi^i, \phi^{*\bar{i}}) = K^{i\bar{i}}\partial_i W\, \bar{\partial}_{\bar{i}}\overline{W}\,. \tag{1.15}$$

The scalar potential is clearly positive definite, and for unbroken supersymmetry the auxiliary components F^i are set to zero, and the potential vanishes identically, consistently with the statement above that supersymmetric solutions have a vanishing cosmological constant.

Non-renormalization theorems in supersymmetric field theory

An extremely powerful feature of supersymmetry is that the superpotential is not renormalized at any order in perturbation theory. This can be seen by 'spurion' analysis, in which one regards the coupling constants in the superpotential as the scalar vacuum expectation values of some non-dynamical background fields. These fields must appear holomorphically in the superpotential and will furthermore be subject to certain super-selection rules. Examples and details are given in e.g. [8], here, let us just exemplify this line of reasoning by considering a superpotential, $W = \frac{\mu}{2}\,\Phi^2$, with a single chiral superfield, Φ, with a dimension–one supersymmetric mass, μ. Treating the coupling constant as a spurion, we note that it can be assigned the charge -2 under a $U(1)$-symmetry under which Φ has unit charge. Furthermore, the superpotential has $U(1)_R$ charge +2, which can be attributed to the field Φ alone, which then has R-charge +1. Thus, we have two algebraic equations constraining any effective superpotential at any energy (under the assumption that it is described by a single chiral superfield): the $U(1)$ global symmetry, and the (also $U(1)$) R-symmetry. Satisfying charge conservation in the effective theory requires the effective superpotential to be of the form $W_{eff} = \frac{\mu}{2}\Phi^2$, which can be matched to the ultra-violet theory in perturbation theory. The only difference between the ultra-violet theory and the low-energy effective theory will then be the renormalization of the kinetic term, which induces a logarithmic — as opposed to polynomial — running of the physical mass-term.

Similar non-renormalization theorems can be proven in string perturbation theory, where an axion pairs up with the universal volume modulus in a chiral multiplet in the four-dimensional supersymmetric effective theory. The axion transforms under a shift symmetry which is unbroken in perturbations theory, which together with holomorphy leads to a non-renormalization theorem for this chiral multiplet [13].

Gauge fields and non-perturbative effects

Gauge multiplets can be included in the theory by using real vector multiplets, which in the Wess-Zumino gauge can be written as,

$$V^a = \theta\sigma^\mu\bar{\theta}\, A^a_\mu + \theta^2\bar{\theta}\,\lambda^{\dagger a} + \bar{\theta}^2\theta\,\lambda^a + \frac{1}{2}\theta^2\bar{\theta}^2\, D^a\,, \tag{1.16}$$

where A^a_μ is a non-abelian gauge potential in the adjoint representation of the gauge group, and λ^a denotes its fermionic superpartner, the gaugino, and D^a is an auxiliary field. Under generalized gauge transformations by the chiral superfield Λ^a, the vector multiplet transforms as $e^{T^aV^a} \to e^{T^a\Lambda^{\dagger a}}e^{T^aV^a}e^{T^a\Lambda^a}$. For practical applications, it is often convenient to work directly with the field strength chiral superfield,

$$\mathcal{W}^a_\alpha = -i\lambda^a_\alpha(y) + \theta_\alpha D^a(y) - (\sigma^{\mu\nu}\theta)_\alpha F^a_{\mu\nu}(y) - \theta^2\sigma^\mu D_\mu\lambda^{\dagger a}\,, \tag{1.17}$$

where $\sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu)$, and D_μ denotes the ordinary covariant derivative.

The super Yang-Mills action can be written in superspace as,

$$\begin{aligned}\mathcal{L}_{SYM} &= \frac{i}{16\pi}\int d^2\theta\,\tau\,\mathcal{W}^a_\alpha\mathcal{W}^a_\alpha + h.c. \\ &= -\frac{1}{4g^2}F^{a\mu\nu}F^a_{\mu\nu} - \frac{\theta_{YM}}{32\pi^2}\,F^{a\mu\nu}\tilde{F}^a_{\mu\nu} + \frac{i}{g^2}\lambda^{\dagger a}\overline{\sigma}^\mu D_\mu\lambda^a + \frac{1}{2g^2}D^aD^a\,,\end{aligned} \tag{1.18}$$

where $\tau = \frac{\theta_{YM}}{2\pi} + \frac{4\pi i}{g^2}$, is the holomorphic gauge kinetic function, including the gauge coupling and the θ-angle. The gauge coupling is clearly marginal at tree-level, but at one loop the gauge kinetic function is renormalized, and can be written as,

$$\tau_{1-loop} = \frac{\theta_{YM}}{2\pi} + \frac{4\pi i}{g^2(\mu)} = \frac{b}{2\pi i}\ln\Big(\frac{\Lambda}{\mu}\Big), \tag{1.19}$$

where $\Lambda = |\Lambda| \,\exp\big(\frac{i\theta_{YM}}{b}\big)$ is the 'holomorphic scale', and $b = 3N - F$ for $SU(N)$ Yang-Mills theory with F flavors. As for the superpotential, the holomorphic gauge kinetic function also abides a certain non-renormalization theorem, ensuring that the one-loop contribution is in fact the *only* contribution to the running of the gauge coupling constant to all orders in perturbation theory. Non-perturbatively, the gauge kinetic function is renormalized by instantons, and may in general be written as,

$$\tau = \frac{b}{2\pi i}\ln\Big(\frac{\Lambda}{\mu}\Big) + \sum_{n=1}^{\infty} a_n \Big(\frac{\Lambda}{\mu}\Big)^{bn}, \tag{1.20}$$

exactly, for some constants b, and a_n.

Charged matter in chiral multiplets transform under gauge transformations as,

$$\Phi \to e^{gT^aV^a}\Phi\,, \tag{1.21}$$

and gauge invariance of the kinetic terms require $K = K(\Phi^\dagger, e^{gT^aV^a}\Phi)$. Charged matter leads to a D-term potential in addition to the F-term potential (1.15), which for canonical kinetic terms is given by,

$$V_D = \sum_a D_a^2\,, \tag{1.22}$$

where $D_a = \sum_i \phi^{*\,i} T^a \phi$, and the sum over a is over gauge group generators, and the sum over i is over the correspondingly charged multiplets.

Furthermore, non-perturbative effects such as gauge instantons or gaugino condensation can induce a non-perturbative contribution to the superpotential of the form,

$$W_{eff} = a\mu^3 \; e^{2\pi i\tau/N} \,. \tag{1.23}$$

These non-perturbative contributions to the superpotential will be important when discussing moduli-stabilization in type-IIB string theory.

Four-dimensional $\mathcal{N} = 1$ supergravity

Supersymmetry is clearly a very beautiful symmetry with the power of turning un-natural operators into technically natural ones, however, as we have presented it in so far, it is a *global* symmetry, requiring the supersymmetry transformations to act the same way on any field through the entire space-time. Experience from particle physics on the other hand suggests that global symmetries in nature are *accidental*, in the sense that they happen to be obeyed by the most relevant operators in the theory, but are fundamentally broken. This point of view is corroborated if dynamical gravity is included, as it is common lore that gravity does not obey any global symmetries — a point which also holds true for string theory. Turning to supersymmetry into a local symmetry is the topic of supergravity, to which we now turn. Supergravity is well-motivated also for phenomenological reasons: the non-observance of a massless Goldstino from spontaneous supersymmetry breaking suggests that — if supersymmetry is relevant for our world — the Goldstino has been 'eaten' in a super-Higgs mechanism, which requires gravity. Furthermore, important phenomenological questions such as supersymmetry breaking and cosmic inflation requires the consideration of gravitation, and the effective theories derived from string theory that we will discuss are all supergravities. Therefore, we will give a brief review of supergravity for the purpose of introducing the notation and basic concepts on which the latter Chapters of this thesis rely. For a full treatment of the basics of supergravity, see [9].

In $\mathcal{N} = 1$ supergravity in four dimensions, the massless graviton, here represented by the *vielbein* e^a_μ, is complemented with a helicity 3/2 fermion called the gravitino, ψ^μ_α. For an off-shell description in the superspace formalism, additional auxiliary degrees of freedom in the form of two vectors need to be included. We will briefly mention the off-shell, conformal compensator formalism of supergravity in Chapter 2, while here we will restrict our attention to the physical component fields.

As in global supersymmetry, the scalar potential for the lowest component of a chiral superfield has two possible contributions: the F-term potential,

$$V_F = e^{K/M_{Pl}^2}\left(K^{a\bar{b}} D_a W\, \overline{D}_{\bar{b}}\overline{W} - 3\left|\frac{W}{M_{Pl}}\right|^2\right), \tag{1.24}$$

and the D-term potential,

$$V_D = \frac{1}{2}\sum_i g_i^2 D_i^2 \,. \tag{1.25}$$

In the F-term potential, the globally supersymmetric contribution is supplemented by M_{Pl}-suppressed terms through the Kähler covariant derivative, $D_a W = \partial_a W + \frac{1}{M_{Pl}^2} K_a W$, and the negative definite contribution proportional to the gravitino mass, $m_{3/2} = e^{K/2M_{Pl}^2}\left|\frac{W}{M_{Pl}^2}\right|$. Kähler transformations are given by,

$$K \;\rightarrow\; K + f(\Phi) + f^*(\bar{\Phi})\,, \tag{1.26}$$

$$W \;\rightarrow\; e^{-f/M_{Pl}^2} W\,. \tag{1.27}$$

The D-term potential is specified in terms of the D-terms for the gauge groups, $D_i = \phi^a T^i K_a + \xi_i$, where ϕ^A is the scalar component of Φ^A, and ξ^i is the corresponding field dependent Fayet-Iliopoulos term. In this thesis, we will focus on the F-term potential, and refer the interested reader to references [14] for a recent discussion on the consistency of the D-term potential in supergravity.

1.2.3 Effective field theories from string theory

String theory is a theory of quantum gravity which reduces to ordinary quantum field theory at low energies. For phenomenology, this is a mixed blessing — while the theory does not necessarily predict any spectacular features at low energies that are incompatible with field theory, the compatibility with the effective field theory formalism is helpful in constructing controlled models of low-energy physics. In the subsequent chapters of this thesis, we will discuss some aspects of ultraviolet sensitivity of the theory which still provide a link between compactifications of string theory and phenomenology. References for this section include [15, 16, 17, 18, 19].

The string S-matrix

In the path integral formalism of quantum mechanics, the partition function can be viewed as a sum over histories, or world-lines in spacetime, of each particle in the theory. Similarly, first quantized string theory can constructed from a path integral over string histories — or world- sheets — through the Polyakov path integral as,

$$\mathcal{Z} = \sum_{\text{world-sheets}} \int \frac{DX\, Dg}{V(\text{diff} \times \text{Weyl})}\, e^{-(S+\lambda\chi)} . \tag{1.28}$$

Here, DX denotes the functional integral over space-time embeddings of the string world-sheet, and Dg denotes the functional integral over world-sheet metrics. The partition function thus denotes the sum over connected world-sheets of different topologies, and each topology is embedded in space-time by X^μ, and weighted by the action of the embedding and the exponential of the Euler number of the string topology, χ. This topological weight can be understood as arising from the two-dimensional Einstein-Hilbert action, which in two dimensions is non-dynamical and serves as a loop counting parameter in the theory. The corresponding coupling constant, $\exp(\lambda)$, is given by the string coupling which is dynamically determined in the theory. For the bosonic string, the action is given by,

$$S = \frac{1}{2\pi\alpha'} \int d^2\sigma \ \sqrt{g}\ g^{ab} \partial_a X^\mu \partial_b X^\nu \ G_{\mu\nu}\,, \tag{1.29}$$

where $\mu,\ \nu$ are a space-time indices, $G_{\mu\nu}$ is the space-time metric, and $a,\ b$, and g_{ab} are the corresponding quantities on the world-sheet. The theory is diffeomorphism and Weyl invariant on the world-sheet, so the path integral is divided by the volume of these symmetries to account for over-counting. In fact, as in quantum field theory, concrete computations requires the fixing of these symmetries through the introduction of Faddeev-Popov ghosts in the path integral, as is discussed in e.g. [15].

In this formalism, S-matrix elements of asymptotic states are, schematically, given by,

$$S_{j_1 \dots j_k}(k_1, \dots, k_k) = \sum_{\Sigma} \int \frac{DX\ Dg\ e^{-(S+\lambda\chi)}}{V(\text{diff} \times \text{Weyl})} \left(\prod_{l=1}^{k} \int d^2\sigma_l\ \sqrt{g(\sigma_l)}\ \mathcal{V}_{j_l}(k_l, \sigma_l) \right), \tag{1.30}$$

where $\mathcal{V}_{j_l}(k_l, \sigma_l)$ denotes the vertex operators of the states in the S-matrix, and the sum over Σ is a short-hand for the sum over the relevant world-sheet topologies.

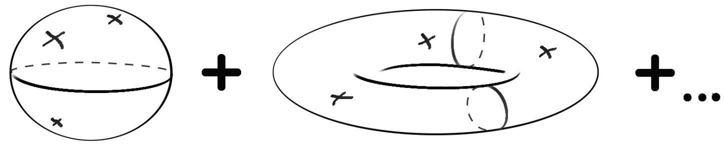


Figure 1.1: The string S-matrix elements are computed in a topological expansion of the world-sheet, here showing the first two orders (corresponding to $g = 0$ and $g = 1$) in a closed string expansion with three vertex operators.

For flat space-time, $G_{\mu\nu} = \eta_{\mu\nu}$, the action (1.29) is that of D non-interacting, massless scalar fields. More general space-time metrics with radius of curvature r can be expanded in the so called sigma model expansion, for which the loop counting parameter is α'/r^2.

Three ways to derive effective supergravities

Effective supergravities can be derived from string theory in at least three ways: from consistency conditions in the world-sheet theory, from scattering amplitudes, and — when applicable — from supersymmetry.

Consistency conditions on the world-sheet string theory can give rise to equations which can be interpreted as effective space-time equations of motion. To see this, consider the contributions to the Weyl anomaly of the bosonic theory at linear order in α' [19],

$$\langle T^a{}_a \rangle = -\frac{1}{2\pi\alpha'}\,\beta^G_{\mu\nu}\, g^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{i}{2\alpha'}\beta^B_{\mu\nu}\,\epsilon^{ab}\,\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\beta^\Phi\, R^{(2)}\,, \qquad (1.31)$$

with,

$$\beta^G_{\mu\nu} = \alpha' R_{\mu\nu} - \frac{\alpha'}{4} H_{\mu\lambda\rho} H_\nu{}^{\lambda\rho} + 2\alpha' \nabla_\mu \nabla_\nu \Phi \,, \tag{1.32}$$

$$\beta^\Phi = \frac{c^{\text{tot}}}{6} - \frac{\alpha'}{2} \nabla^2 \Phi + \alpha' \nabla_\rho \Phi \nabla^\rho \Phi - \frac{\alpha'}{24} H_{\mu\nu\rho} H^{\mu\nu\rho} \,, \tag{1.33}$$

$$\beta^B = -\frac{\alpha'}{2} \nabla^\rho H_{\rho\lambda\nu} + \alpha' (\nabla^\rho \Phi) H_{\rho\mu\nu} \,. \tag{1.34}$$

Here Φ is the dilaton,[3] and $H_{\mu\nu\rho}$ is an anti-symmetric three-tensor. Furthermore, the zeroth order contribution to the β-functions in the α'-expansion is proportional to c^{tot}, which in a D space-time dimensional bosonic theory with a '*bc*' CFT on the world-sheet is equal to $D - 26$. Consistency requires that all β-functions vanish, why the zeroth order contribution in the α'-expansion implies that $D = 26$, and higher orders specify the equations of motion for the fields in the effective low-energy theory. To linear order in α', these equations include the Einstein equation for the space-time metric, while higher order terms provide corrections to these equations. A field theory that reproduces these equations to linear order in α' and satisfies the condition $D = 26$ is given by,

$$S_{eff} = \frac{1}{2\kappa^2} \int d^{26}x \ \sqrt{G}\, e^{-2\Phi} \left(R - \frac{1}{12} H_{\mu\nu\lambda\rho} H^{\mu\nu\lambda\rho} + 4\partial_\mu \Phi \partial^\mu \Phi \right) , \tag{1.35}$$

which is the effective action of bosonic string theory with a cut-off scale of $\Lambda_0 = M_{string}$ to leading order in α'. We note that quantum loop computations in this field theory matches the quantum string loop expansion.

Effective supergravities can also be derived in a 'coupling-by-coupling' fashion by matching the amplitudes of an effective field theory to the string theory computation of the same coupling. In Chapter 2, we will briefly allude to how this method can be used to understand the effects of moduli stabilization on the spectrum of models of particle physics with broken supersymmetry.

[3]The string coupling is given by e^Φ.

Type II supergravity in ten dimensions

Finally, in ten-dimensional[4] superstring theory with maximal space-time supersymmetry, i.e. 32 supercharges in ten dimensions, one can show that there are only two possible effective supergravities,[5] and by inspection of the low-energy spectrum of each of the string theories, one can easily identify the correct effective theory. These supergravities, called type IIA and type IIB supergravity, respectively, are given by the actions,

$$\begin{aligned} S_{IIA} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\ \sqrt{-G}e^{-2\Phi}\left(R+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}|H_3|^2\right) \\ &- \frac{1}{4\kappa_{10}^2}\int d^{10}x\ \sqrt{-G}\left(|F_2|^2+|\tilde{F}_4|^2\right)-\frac{1}{4\kappa_{10}^2}\int d^{10}x\, B_2\wedge F_4\wedge F_4 \\ &+ \ \text{fermions}\,, \end{aligned} \tag{1.36}$$

where in the NS-NS sector, Φ is the dilaton, B_2 is an antisymmetric two-tensor, and $H_3 = dB_2$. In the R-R sector, the IIA theory contains the potentials C_1 and C_3, from which the two-form and four-form fluxes $F_2 = dC_1$, and $F_4 = dC_3$ can be constructed. Here, $d\tilde{F}_4 = F_4 - C_1 \wedge H_3$. This field content corresponds to the massless states of type IIA string theory, and is therefore only valid up to energies of M_{string}, above which the full string theory must be considered.

The type IIB supergravity action has the same NS-NS sector as type IIA, but differs in the R-R sector, which in IIB involves the gauge potentials C_0, C_2, and

[4]For the superstring, for the left-moving and right-moving sectors separately, $c^{tot} = c_{X,\psi} + c_{ghost} = c_{X,\psi} - 15$, which implies that the theory of bosons need $c_{X,\psi} = 15$. Since each bosons contribute with one, and each fermion with 1/2 to the central charge, this condition can be satisfied by taking the space-time dimension to be ten for both left- and right movers.

[5]The uniqueness follows from restricting the supergravities to not involve higher spin fields, which are problematic to couple to gravity.

C_4. The ten-dimensional action is given by,

$$\begin{aligned} S_{IIB} &= \frac{1}{2\kappa_{10}^2} \int d^{10}x \ \sqrt{-G} e^{-2\Phi} \left(R + 4\partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} |H_3|^2 \right) \\ &- \frac{1}{4\kappa_{10}^2} \int d^{10}x \ \sqrt{-G} \left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2} |\tilde{F}_5|^2 \right) \\ &- \frac{1}{4\kappa_{10}^2} \int d^{10}x \, C_4 \wedge H_3 \wedge F_3 + \text{fermions} \, , \end{aligned} \tag{1.37}$$

where $\tilde{F}_3 = dC_2 - C_0 \wedge H_3$, and $\tilde{F}_5 = dC_4 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$. In fact, the above action must be supplemented with a self-duality condition on the five-form, $\tilde{F}_5 = \star \tilde{F}_5$. The action (1.37) is given in the 'string frame' and does not have canonically normalized gravitational terms. A simple re-definition of the metric, $g_{E\,MN} = e^{-\Phi/2} G_{MN}$, puts the action in the 'Einstein frame', with the standard Einstein Hilbert term[6],

$$\begin{aligned} S_{IIB} &= \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left\{ R - \frac{\partial_M \tau \partial^M \bar{\tau}}{2(\text{Im}\,\tau)^2} - \frac{G_3 \cdot \bar{G}_3}{12 \text{Im}\,\tau} - \frac{\tilde{F}_5^2}{4 \cdot 5!} \right\} \\ &- \frac{i}{8\kappa_{10}^2} \int \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\text{Im}\,\tau} \, , \end{aligned} \tag{1.38}$$

where $G_3 = F_3 - \tau H_3$, and $\tau = C_0 + ie^{-\Phi}$.

The low-energy D-brane action

D-branes appear in string perturbation theory as objects onto which open strings can end, and are dynamical objects in the low-energy theory. The world-volume action for a Dp-brane contains two parts: the first corresponds to a generalization of the Born-Infeld non-linear action, initially suggested for electrodynamics, while the second part encodes the coupling between a D-brane and the RR-forms to which it appears 'electrically' charged. In more detail,

[6]In this form, the action has a manifest $SL(2, \mathbb{Z})$ symmetry.

$S = S_{DBI} + S_{CS}$, where for a stack of p-branes [20],

$$S_{DBI} = -T_p \int d^{p+1}\sigma \,\mathrm{tr}\left(e^{-\phi}\sqrt{-\det(P[E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij}E_{jb}] + \lambda F_{ab})\ \det Q^i_j}\right) \quad (1.39)$$

where $E_{ab} = G_{ab} + B_{ab}$, $Q^i_j = \delta^i{}_j + i\lambda[\Phi^i, \Phi^k]E_{kj}$, and P denotes the pull-back onto the world-volume of the D-brane stack. The Chern-Simons can in general be written as,

$$S_{CS} = -\mu_p \int \mathrm{tr}\left(P[e^{i\lambda\, \mathbf{i}_\Phi \mathbf{i}_\Phi}\left(\sum C^{(n)} e^B\right)]\ e^{\lambda F}\right), \quad (1.40)$$

where $\mathbf{i_\Phi}\, C^{(p)} = \frac{1}{(p-1)!}\,\Phi^\mu C^{(p)}_{\mu\nu_1\ldots\nu_{p-1}}$. Turning off the world-volume flux and NS-flux, and restricting to a single Dp-brane, this action simplifies to,

$$S_{DBI} = -T_p \int d^{p+1}\sigma\, e^{-\phi}\sqrt{-\det\left(P[-G]\right)}, \quad (1.41)$$

$$S_{CS} = -\mu_p \int P[C^{(p+1)}]. \quad (1.42)$$

A chiral spectrum for a stack of branes can be obtained by placing the branes at singular points in the internal geometry, or by turning on fluxes on the world-volume of the branes.

Compactification and moduli

In order to connect the above theories to the apparently four-dimensional real world, the extra dimensions need to be compactified, i.e. the ten dimensional integral in the action is taken over a space which has some product structure, $M_{10} = M_4 \times M_6$, where, in order to describe our universe, M_4 should be well approximated by Minkowski space, and M_6 should at least be small enough to have avoided detection as until now. Compactifications have the further virtue of breaking some of the supersymmetry, e.g. type II theories compactified on a Calabi-Yau manifold preserves an $\mathcal{N} = 2$ supersymmetry in four dimensions.

While four dimensional $\mathcal{N} = 2$ supergravity in four dimensions is still too restrictive to allow for even semi-realistic phenomenology, understanding this type of compactifications is important in order to understand a class of more phenomenologically successful compactifications with $\mathcal{N} = 1$ supersymmetry, which are constructed from orientifold compactifications of type IIB theory. In string theory, there also exists a number of other ways of obtaining $\mathcal{N} = 1$ in four dimensions, including type IIA orientifolds, M-theory on G_2-manifolds, and heterotic compactifications.

With this motivation, we here review the basics of compactifications of ten-dimensional supergravity on six-dimensional manifolds in the absence of flux, i.e. $F_5 = F_3 = F_1 = H_3 = 0$ in type IIB, and $F_4 = F_2 = H_3 = 0$ in type IIA. Compactifications which preserve some supersymmetry in this background (with $\partial_\mu \Phi = 0$), can be shown admit a covariantly constant spinor, which is equivalent to requiring the compactification manifold to have $SU(3)$ holonomy and a vanishing first Chern class. By Yau's theorem, such a Kähler manifold admits a Ricci-flat metric. The spectrum of the four-dimensional theories compactified on the Calabi-Yau can be obtained by dimensional reduction, i.e. by integrating out the dependence on the internal dimensions. Schematically, a p-form potential A_p (here representing the potentials in either sector) satisfies the ten-dimensional equation of motion,

$$d_{10} \star_{10} d_{10} A_p = 0 \,, \tag{1.43}$$

where we have neglected possible source terms coming from the Chern-Simons action. For a more detailed discussion, we refer the reader to [16]. In a gauge in which $d^\dagger A_p = 0$, equation (1.43) implies that,

$$\Delta_{10} A_p = (\Box_4 + \Delta_6) \; A_p = 0 \,, \tag{1.44}$$

where we have decomposed the Laplace equation onto Minkowski space and the internal manifold, while consistently assuming the metric to be separable. From equation (1.44), it is evident that an expansion of A_p in eigenmodes of Δ_6 gives a Kaluza-Klein tower of four dimensional modes with masses corresponding to their eigenvalue of Δ_6. The mass scale of these modes is set by the compactification scale, $\sim (1/\mathrm{Vol}_{\mathrm{CY}})^{1/6}$, and we can consider an effective theory below the compactification scale in which only the massless modes are included. Geometrically, this corresponds to the modes of A_p which are harmonic on the compactification space. By Hodge's theorem, the space of these modes is isomorphic to the homology space $H^p(M_6)$ of closed up to exact p-forms. By being able to relate the massless spectrum to topological (Hodge) numbers, much can be said about the low-energy physics without knowing the details of the Calabi-Yau geometry.

The metric can be perturbed without changing the supersymmetry condition, $R_{mn}(g + \delta g) = 0$, for $m,\ n$ being indices on the internal space, as long as the metric perturbations, δg satisfy the Lichnerowicz equation, which on a Kähler manifold reduces to requiring that δg is harmonic. It can be shown that these metric perturbations separate into $h^{2,1}$ 'shape' perturbations,

$$\Omega_{abc} g^{c\bar{d}} \delta g_{\bar{d}\bar{e}}\, dz^a \wedge dz^b \wedge d\bar{z}^{\bar{e}}\,, \tag{1.45}$$

and $h^{1,1}$ 'volume' perturbations,

$$\delta g_{a\bar{b}}\, dz^a \wedge d\bar{z}^{\bar{b}}\,, \tag{1.46}$$

and the corresponding moduli are called *complex structure* and *Kähler*, respectively.

To leading order in α' (and in the absence of D-branes), the Kähler metric on the moduli space factorizes on the complex structure and Kähler moduli spaces, and the Kähler potential is given by,

$$K = K_{C.S.} + K_K = -\ln\left(i\int_M \Omega\wedge\bar{\Omega}\right) - -\ln\left(\frac{4}{3}\int_M J\wedge J\wedge J\right), \tag{1.47}$$

where J is the Kähler form and Ω is the holomorphic three-form on the moduli space [21].

All things considered, the closed string spectrum of type IIB theory compactified on a Calabi-Yau three-fold consists of a gravity multiplet, $h^{1,1}+1$ hypermultiplets and $h^{2,1}$ vector multiplets. Type IIA compactified on a Calabi-Yau has, apart from the gravity multiplet, a spectrum consisting of $h^{1,1}$ vector multiplets and $h^{2,1}+1$ hypermultiplets [17].

Let us summarize: after Calabi-Yau compactification, we have arrived at a theory which do not yet contain the Standard model, which have $\mathcal{N}=2$ supersymmetry and are therefore non-chiral, and have a typically large number of massless moduli which only couple gravitationally to matter. All these things are phenomenologically problematic, and we will now turn to the issue of stabilizing the moduli in type II theory.

1.2.4 Moduli stabilization and the landscape of string vacua

We have reviewed how controlled string theory compactifications all come with light massless degrees of freedom which only couple to matter gravitationally. Naively, one might expect that constraints on this type of matter to be weak, since the coupling to any measurement apparatus is very small. However, the non-observance of any 'fifth forces', as well as cosmological constraints on light matter restrict the allowed spectrum of moduli, which is typically taken to mean that the moduli-masses should be of the order of 10 TeV or heavier. It is therefore necessary to generate a potential for the moduli in such a way that they become stabilized with non-vanishing masses.

However, it has been long known that stabilization of the moduli can be quite complicated, as was illustrated for the universal volume modulus by a simple argument by Dine and Seiberg [22]: in all compactifications, there exists a decompactification limit where the potential goes to zero as the volume modulus, ρ, diverges. The regime of large ρ is also the regime of weak coupling, and under the assumption that quantum corrections are necessary to generate contributions to the potential with different volume scaling, a meta-stable solution at finite values of ρ necessarily involves quantum corrections to the potential. However, if the first few quantum corrections are important enough to change the structure of the potential, it is typically not consistent to truncate the expansion at a finite order, and the potential must be computed to all orders to find a well-controlled solution. The Dine-Seiberg problem can be avoided in certain scenarios, but it still serves as a reminder that there is no guarantee that the four-dimensional, low-energy effective description is related in weak coupling to the ultraviolet complete theory.

The 'no-go' theorem of Maldacena and Nũnez

Furthermore, a celebrated theorem by Maldacena and Nũnez shows that compactifications to maximally symmetric four-dimensional spaces (i.e. Minkowski, anti-de Sitter or de Sitter) necessarily are quite complicated [23]. The 'no-go' theorem considers a general D-dimensional theory of Einstein-Hilbert gravity which includes massless scalar fields, such as could arise from e.g. p-form fluxes, with arbitrary couplings, positive kinetic terms, and zero or negative potential. The theorem states that, except if there are fluxes with $p = 1$ or $p = D-1$, there are *no warped* compactifications to de Sitter space or Minkowski space of the form,

$$ds_D^2 = \Omega(y)\left(ds_4^2(x) + ds_{D-4}^2(y)\right) , \tag{1.48}$$

where y is the coordinate on the internal space (which is assumed to be compact), and the warp-factor $\Omega(y)$ is assumed not to diverge anywhere. If $p = 1$ or $p = D - 1$, compactifications to Minkowski space, but not de Sitter, are possible, with a trivial, constant, warp factor.

To understand the implications of this, let us exemplify the theorem by considering warped compactifications to Minkowski space of type IIB supergravity in which the Einstein metric takes the form,

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}(y)dy^m dy^n , \tag{1.49}$$

the five-form flux is given by,

$$\tilde{F}_5 = (1 + \star_{10})\, d\alpha(y) \wedge d\widehat{\mathrm{vol}}_4 , \tag{1.50}$$

where $d\widehat{\mathrm{vol}}_4$ is the Minkowski space volume form, and the three-form flux, G_3, is assumed not to break four-dimensional Poincaré invariance. In general, the axio-dilaton τ may vary holomorphically in the internal space: $\tau = \tau(y)$.

With this Ansatz, the trace-reversed external components of the Einstein equation are given by,

$$\tilde{\nabla}^2 e^{4A(y)} = e^{2A(y)}\frac{G_{mnp}\bar{G}^{mnp}}{12\mathrm{Im}\,\tau} + e^{-6A}\left[\partial_m\alpha\partial^m\alpha + \partial_m e^{4A}\partial^m e^{4A}\right] , \tag{1.51}$$

as reviewed in e.g. [24]. Now, the theorem of Maldacena and Nũnez — adapted to this specific context — is based on noticing that the integral of the left hand side of equation (1.51) over the internal space vanishes, while the right hand-side only has positive definite contributions. The only consistent solution to this equation is then, e^{4A} = constant, α = constant, and $G_{mnp} = 0$, which is a direct product compactification without flux.

Flux compactifications

String theory manages to avoid the conclusions of the 'no-go' theorem by not quite satisfying the assumptions of pure supergravity. On top of the IIB action, string compactifications allow for localized sources in the form of D-branes and O-planes, and α'-corrections to the Einstein-Hilbert action. The impact of the Maldacena-Nũnez theorem is therefore *not* to show that compactifications to de Sitter or Minkowski vacua are impossible, but rather that they necessarily are quite complicated. Giddings, Kachru and Polchinski (GKP) famously considered warped compactifications to Minkowski space in string theory, and found that upon inclusion of an action for localized sources, $S_{tot} = S_{IIB} + S_{loc.}$, the external Einstein equations are given by,

$$\tilde{\nabla}^2 e^{4A(y)} = e^{2A(y)} \frac{G_{mnp}\bar{G}^{mnp}}{12\mathrm{Im}\,\tau} + e^{-6A}\left[\partial_m\alpha\partial^m\alpha + \partial_m e^{4A}\partial^m e^{4A}\right] + \frac{\kappa_{10}^2}{2} e^{2A}(T^m_{\ m} - T^\mu_{\ \mu})^{loc.}, \tag{1.52}$$

where $T^{loc.}_{MN} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{loc.}}{\delta g^{MN}}$ [24]. The flux may be quantized as,

$$\int_{M_q} F_q = N\left(2\pi\sqrt{\alpha'}\right)^{q-1}. \tag{1.53}$$

For a p-brane wrapped on a $p-3$-cycle Σ, the contribution from the localized source is given by,

$$(T^m_{\ m} - T^\mu_{\ \mu})^{loc.} = (7-p)T_p\,\delta(\Sigma)\,. \tag{1.54}$$

A negative contribution can come from $p < 7$ and $T_p < 0$, which is the case for e.g. O3-planes. After inclusion of the leading α'-corrections to the Chern-Simons action, it can be shown that wrapped D7-branes also give a negative contribution to the right hand-side of equation (1.54). Giddings, Kachru and Polchinski furthermore noted that in the special case that all local sources *saturates* the inequality

$$(T^{m}_{\ m} - T^{\mu}_{\ \mu})^{loc.} \geq 4T_3\rho_3^{loc.}\,, \tag{1.55}$$

the form of the solution is completely determined, and is given by an imaginary self-dual three-form (ISD) flux,

$$\star_6\, G_3 = iG_3\,, \tag{1.56}$$

and the warp factor that is related to the five-form flux as,

$$e^{4A(y)} = \alpha(y)\,. \tag{1.57}$$

The ISD-condition on the three-form flux requires G_3 only to be a linear combination of primitive $(2,1)$-flux and $(0,3)$-flux, and the solution is supersymmetric only if the $(0,3)$ part vanishes.

At the level of the four-dimensional effective action, the inclusion of the non-trivial three-form flux gives rise to a Gukov-Vafa-Witten superpotential [25],

$$W = \int \Omega \wedge G_3\,, \tag{1.58}$$

where Ω is the holomorphic three-form on the compactification manifold. The Kähler potential is given by,

$$K = -3\ln\left(-i(\rho - \bar{\rho})\right) - \ln\left(-i(\tau - \bar{\tau})\right) - \ln\left(-i\int \Omega \wedge \bar{\Omega}\right). \tag{1.59}$$

The superpotential (1.58) can supersymmetrically stabilize the complex structure moduli and the dilaton — but not the Kähler moduli — and the corresponding model is of *no-scale* type, in which the overall volume is a flat direction.

A number of more or less successful variants of the GKP-compactifications have been suggested in order to stabilize the Kähler moduli as well as the complex structure moduli, and here we will briefly review two of the most important scenarios: the KKLT-scenario [26], and the Large Volume Scenario [27].

The KKLT scenario

The no-scale structure of GKP-compactifications results in solutions in which the overall volume modulus — and all other Kähler moduli — are flat directions in the low-energy theory to lowest order in g_s and α'. Corrections to the scalar potential will generically break the no-scale structure and stabilize the Kähler moduli, and obtaining reliable and computable models for how this happens is crucial for understanding the low-energy physics.

The KKLT scenario, [26], achieves this breaking by working with relatively large volume in string units, and by invoking non-perturbative effects to stabilize the Kähler moduli. As in reference [26], we will here confine our discussion to the case of a single Kähler modulus, denoted ρ, and flux compactifications in which there are no light D-brane degrees of freedom.[7] With these assumptions, the four-dimensional low-energy theory only contains the light volume modulus chiral superfield with a Kähler potential,

$$K = -3\ln(-i(\rho - \bar{\rho}))\,, \tag{1.60}$$

and, at tree-level and to leading order in α', a constant superpotential $W = W_0$. More generally, the Kähler potential will receive no-scale breaking perturbative corrections in g_s and α', and the superpotential will receive non-perturbative

[7] In §2, we will discuss a version of this scenario in which light D3-brane moduli are included in the low-energy effective theory.

corrections involving ρ. A controlled regime can be obtained if the volume is sufficiently large with respect to the string scale, which corresponds to tuning the constant W_0 to be smaller than its natural value at the flux scale. In this regime, the corrections to the Kähler potential — which are volume suppressed — can controllably be made smaller than the non-perturbative corrections to the superpotential, and we can consistently include only the latter type of no-scale breaking corrections in the effective theory [26].

The non-perturbative corrections to the superpotential may arise from either Euclidean-D3-branes, in which case,

$$W_{np} = A\, e^{2\pi i\rho}\,, \tag{1.61}$$

or from gaugino condensation in the gauge theory of a stack of N D7-branes wrapping the four-cycle corresponding to ρ, in which case

$$W_{np} = \Lambda^3 = A\, e^{2\pi i\rho/N}\,. \tag{1.62}$$

In both cases, the value of the pre-factor A typically depends on the vacuum expectation values of the stabilized complex structure moduli. Including corrections of this form, the supersymmetry condition for ρ, $D_\rho W = 0$ implies that,

$$W_{np} = -\frac{3N}{2\pi(\rho - \bar{\rho})}\, W\,, \tag{1.63}$$

where $N = 1$ for non-perturbative effects arising from ED3-branes, and N is equal to the number of D7-branes if gaugino condensation is responsible for the non-perturbative superpotential. The corresponding equation for $\sigma = \mathrm{Im}\,\rho$ is transcendental, and can be solved for large volumes only for $W_0 \ll 1$.

Finally, the authors of [26] argued that de Sitter vacua can be obtained if supersymmetry breaking effects, such as an anti-D3 brane at the bottom of a warped throat.[8] In Chapter 2, we will discuss a simple example of the KKLT scenario more explicitly, and in Chapter 4, we will discuss the typical spectrum of scalar fields in the AdS vacuum.

The scale of supersymmetry breaking in this scenario is $m_{3/2} \approx |W_0|/\mathcal{V}$, where $\mathcal{V}$ is the volume of the internal dimensions in string units. To connect this model to phenomenology, one may embed a visible sector, including the standard model matter spectrum, in a number of different ways, e.g. through branes at singularities or through intersecting branes. We will return to the phenomenology of models of particle physics in §2.

The Large Volume Scenario

As a second example of variations of the GKP-compactifications in which no-scale breaking effects stabilize the volume modulus, let us briefly review the Large Volume Scenario [27], in which the leading α'-corrections to the Kähler potential are included, and a regime in which $W_0 \approx 1$, and the volume $\mathcal{V}_{CY} \gg 1$ in string units. The Kähler potential is in this case given by,

$$K = K_{cs} - 2\ln\left(e^{-3\phi_0/2}\mathcal{V} + \frac{\xi}{2}\left(\frac{-i(\tau-\bar{\tau})}{2}\right)^{3/2}\right), \tag{1.64}$$

where $\xi = -\frac{\zeta(3)\,\chi(M_6)}{2(2\pi)^3}$ [28]. The superpotential is of the form

$$W = W_0 + \sum_n A_n\, e^{ia_n\rho_n}\,. \tag{1.65}$$

Even after integrating out the complex structure moduli and the dilaton — which again are assumed to be supersymmetrically stabilized by three-form flux — the scalar potential for the Kähler moduli is quite complicated, and we refer

[8]The warping is necessary to tune down the energy density of the supersymmetry breaking and to avoid a run-away of the volume modulus.

the reader to reference [27] for the full details. For our purposes it is sufficient recapitulate the main argument: at large volumes of the constituent four-cycle, the potential approaches zero from below, while at smaller volumes, the potential is positive. It follows by continuity that there is a vacuum at intermediary values of the volume, and this intermediary volume can be shown to be quite large in string units.[9] In this scenario, $W_0 \approx 1$, and the resulting scales and spectrum of soft masses are non-trivial, and we will discuss them in more detail in §2.

Bousso-Polchinski and the landscape

Even before the flux compactifications of the GKP form were discovered, it had been noticed that the quantization of flux wrapped on internal cycles in a string theory compactification could have highly interesting physical consequences. For instance, Bousso and Polchinski noted in [29] that a large number of flux-cycles can result in vacua with tightly spaced cosmological constants. To see this, consider a ten-dimensional action including p-form flux,

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \, \sqrt{-g} \left(R - \frac{F_p \cdot F_p}{2p!} + \ldots \right) , \tag{1.66}$$

where the ellipsis may include fermionic terms as well localized sources. The number of non-trivial p-cycles in the internal manifold onto which the p-form flux can be quantized is determined topologically, and is here denoted by J. The flux on cycle i is then by Dirac quantization fully specified by an integer N_i, and after dimensional reduction to four dimensions the effective cosmological

[9] E.g. $\mathcal{V} \approx 10^8$ in string units in the first implementations of the model, however see Chapter §2 for a more detailed discussion of the different versions of the scenario.

constant is given by,

$$\Lambda = \Lambda_0 + \sum_i^J k_i N_i^2 \,, \tag{1.67}$$

where Λ_0 denotes an assumed negative 'bare' cosmological constant, which could arise from curvature in the internal dimensions, or from negative tension objects. The constants k_i depend on the compactification geometry, and since the flux scale is much larger than the scale of the four-dimensional effective cosmological constant, it may appear that obtaining a small effective cosmological constant requires a large fine-tuning of Λ_0 or k_i. However, if the number of flux-cycles is large, the condition $0 < \Lambda < \Delta\Lambda$ for some small $\Delta\Lambda \approx 10^{-120}\ M_{Pl}^4$, amounts to counting the number of lattice points in a J-dimensional shell with radial width given by,

$$\Delta r = \frac{\Delta\Lambda}{2\sqrt{2|\Lambda_0|}} \,. \tag{1.68}$$

The volume of the shell is then $\Delta Vol = \omega_{J-1} r^{J-1} \Delta r$, where $\omega_{J-1} = 2\pi^{J/2}/\Gamma(J/2)$ denotes the volume of the unit $(J-1)$-sphere. The typical number of lattice points in the shell is then,

$$\frac{\Delta Vol}{\prod_i^J k_i} = \frac{\pi\, |2\pi\Lambda_0|^{J/2-1}\, \Delta\Lambda}{\bar{k}^J\, \Gamma(J/2)} \,. \tag{1.69}$$

If the geometric average of k_i, here denoted $\bar{k}$, is not too large, this is a rapidly increasing function of J in the most interesting range of $O(100)$ flux cycles, which, for example, is larger than unity for $J \in [191, 2810]$, for the parameter values $\bar{k} = .1$, $\Lambda_0 = 1$, and $\Delta\Lambda = 10^{-120}$ in natural units. The values of k_i can be computed from the ten-dimensional theory, and are typically proportional to the inverse volume of the compactification in string units, and is therefore appropriately small in well-controlled supergravity compactifications [29].

The resulting distribution of the effective cosmological constant has support for both positive and negative values, and range up to the scale $|\Lambda_0|$. For large enough J, certain choices of flux result in an effective cosmological constant, Λ, at the scale of the observed cosmological constant.

While this pioneering work suggests that a small cosmological constant is possible to accommodate in flux compactifications, it does not explain why in our particular universe, this value should be small. 'Anthropic' arguments similar to the ones used by Weinberg in reference [38] to predict the value of the cosmological constant have been proposed as a key part of the explanation, however, the resulting 'anthropic landscape' relies on assumptions about quantum gravity and the cosmology of the universe, such as eternal inflation. In this thesis, we will not discuss these assumptions any further, but in Chapter 4, we will revisit the question of the apparent abundance of vacua in the low-energy effective theory of flux compactifications.

The statistics of the string landscape

As we have reviewed, the Bousso-Polchinski landscape indicates how the cosmological constant problem may be solved in string theory, and since this pioneering work was published, a number of authors have developed the heuristic picture of [29] into more concrete realizations, while also taking into account the shape of the potential for the moduli fields. Of particular importance are the works of Ashok and Douglas [30], Douglas [31], and Denef and Douglas [32, 33], part of which we will review in Chapter 4.

The statistical distribution of vacua in the low-energy effective theory can, schematically, be found by considering the sum,

$$N_{vac} = \sum_a \sum_i \text{Vacua}_i(\text{Theory}_a)\,, \tag{1.70}$$

where the sum over a is over all effective theories arising from string compactifications, and the sum over i counts the number of meta-stable vacua in each such theory. The different theories enumerated by a differ e.g. by different choices of flux, or by brane content. In the low-energy theory these choices are reflected in different values for parameters of the Lagrangian, and different gauge groups and matter contents. For a supersymmetric theory, here simplified as to involve only chiral superfield, the discrete sum over vacua can be approximated by the continues measure,

$$N_{vac} = \int \mathcal{D}W\mathcal{D}\bar{W}\mathcal{D}K\ f(K, W, \bar{W})\ \delta^{(2N)}(\partial_a V)\ |\det(\partial_{ab} V)|\,, \tag{1.71}$$

where $f(K, W, \bar{W})$ is a measure on the space of effective theories, which in principle could be derived from string theory. Given such an ensemble of vacua, one may condition on values of parameters in the effective theory, such as the cosmological constant, the supersymmetry breaking scale, and the Yukawa couplings, so as to obtain a class of vacua which are compatible with experiments. More details on this methodology can be found in e.g. [31]. A much studied corner of the landscape is given by flux vacua of type IIB, in which statistical studies have predominantly focused on the complex structure moduli, z^a, of the theory. In this case, the superpotential is given by $W = \sum_{i=1}^{K} N^i \Pi_i(z)$ and the Kähler potential is $K(z, \bar{z}) = -\ln(Q^{ij}\Pi_i\bar{\Pi}_j)$, where $\Pi(z)$ is a complicated, but computable, holomorphic function of the moduli, and K denotes the number of flux cycles, $K = 2b_3$. The tadpole condition becomes $\frac{1}{2}Q^{ij}N_iN_j \leq L$, where L depends on the details of the compactification, with typical values in the range $10 - 1000$. An approximate expression for the density of supersymmetric vacua was derived in [32], and is given by,

$$dN_{vac}(z) = \frac{(2\pi L)^{K/2}}{(K/2)!\ \pi^{K/4}}\ \det\left(R(z) + J(z)\right)\ , \tag{1.72}$$

where $R(z) = \frac{i}{2} R^{a}{}_{bc\bar{d}}\, dz^c \wedge d\bar{z}^{\bar{d}}$ is the curvature form on the field space and $J(z) = \frac{i}{2} g_{a\bar{b}} dz^a d\bar{z}^{\bar{b}}$ is the Kähler form on the field space.

1.3 Supersymmetry Breaking and Sequestering

Let us now turn to the particular questions addressed in this thesis, starting with the question of supersymmetry breaking and sequestering in string compactifications, which is discussed in full detail in Chapter 2.

Whether or not supersymmetry is realized close to the electroweak scale is currently a question under experimental investigation, and the LHC experiments ATLAS and CMS have both produced improved and fairly strong bounds on masses of e.g. the strongly coupled gluinos, $m_{\tilde{g}} \gtrsim 500$ GeV [34]. Furthermore, if the tentative evidence for a Higgs boson at around 125 GeV turns into a discover of the particle at these energies, this would impose a strong constraints on some models of supersymmetry breaking, see e.g. [35]. These new constraints are highly interesting for model building, yet they do not mean that supersymmetry is not a part of the solution to the gauge hierarchy problem, since this only requires the superpartner of the top-quark to have a mass of the order of the electroweak scale.

In Chapter 2, we discuss how a different type of experimental bounds, arising from bounds on rare flavor changing processes, can already be used to constrain some models of supersymmetry breaking in string theory. As we will review in great detail in §2.1, well-motivated models of supersymmetry breaking in string compactifications give rise to computable versions of so called 'gravity mediation', in which the soft masses and interactions of the visible sector superpartners arise from operators which can be written in spurion form as,

$$W \supset \frac{a_{ijk}}{3!\, M_{Pl}} X \Phi^i \Phi^j \Phi^k + \frac{\mu_{ij}}{2\, M_{Pl}} X \Phi^i \Phi^j \,, \tag{1.73}$$

$$K \supset \frac{c_{i\bar{j}}}{M_{Pl}^2} |X|^2 \bar{\Phi}^{\bar{i}} \Phi^j + \left(\frac{n_{\bar{i}j}}{M_{Pl}} X \bar{\Phi}^{\bar{i}} \Phi^j + h.c \right) , \tag{1.74}$$

$$f_{ab} \supset \frac{f_{(a)}}{g_a^2\, M_{Pl}} X \,, \tag{1.75}$$

where $X = \theta^2 F$, is the chiral superfield parametrizing the supersymmetry breaking sector, and Φ^i denotes a field in the visible sector. By integrating out the auxiliary fields of the visible sector fields, these terms can be seen to give rise to soft scalar masses, gaugino masses, pure 'holomorphic' masses, and cubic scalar interactions. These terms would all be of the electro-weak scale if $F \approx 10^{21}$ (GeV)^2.

Historically, it has been common to assume that the coefficients of these superpotential and Kähler potential terms are 'flavor blind', and therefore universal, i.e. proportional to the unit matrix in flavor space, however this assumption is in general not well-motivated and natural theoretically. On the other hand, off-diagonal coefficients of say $c_{i\bar{j}}$ are tightly constrained by bounds on flavor changing neutral currents, so if this mediation scenario is realized in nature, these coefficients must be small. This discrepancy between what appears 'generic' in the low-energy effective theory, and the experimental bounds is called the supersymmetric flavor problem.

Clearly, the mediating terms in equations (1.73), (1.74), and (1.75) are non-renormalizable and Planck-mass suppressed, why they according to the general framework of effective field theory discussed in §1.2, should be computed in a theory at the energy-scale M_{Pl}, i.e. in a quantum theory of gravity. This motivates the study of supersymmetry breaking in string theory.

In type II string theory, as well as in M-theory and F-theory, the visible spec-

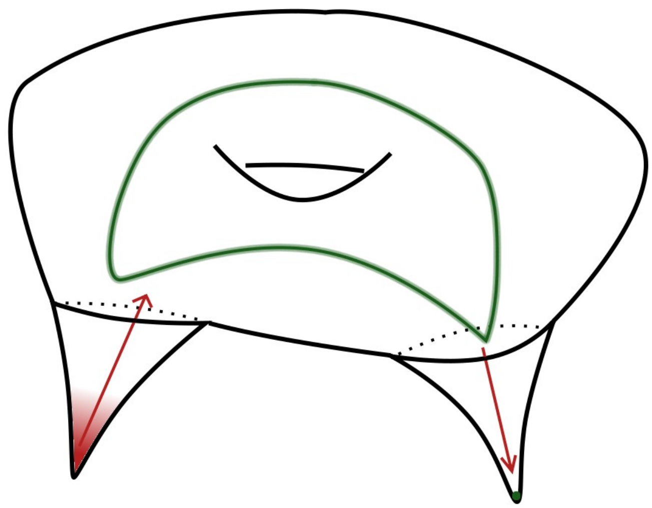


Figure 1.2: The mediation of supersymmetry breaking may proceed through a hidden supersymmetry breaking sector inducing an F-term of the volume modulus, which in turn may induce soft terms in the visible sector.

trum can be located on a stack of branes which are geometrically separated from the visible sector in the extra dimensions, and it has been suggested that such a separation will automatically solve the flavor problem of intermediate-scale mediation [43]. In Chapter 2, we will argue that in stabilized string compactifications, the flavor problem typically re-appears after the stabilization of moduli has been taken into account. The reasoning for this is given pictorially in figure 2.1, which illustrates how a geometrically sequestered supersymmetry breaking sector interacts with the overall volume modulus of the compactification, forcing this modulus to participate in the supersymmetry breaking through a non-vanishing F-term of the corresponding chiral superfield. The volume modulus is prohibited to appear in the superpotential to any order in perturbation theory, but non-perturbative effects can induce cross-couplings between the visible sector and the volume superfield, which in term give rise to soft terms of the form of equations (1.73), (1.75).

In Chapter 2, we consider the consequences of this type of supersymmetry breaking in full detail, and we find that for moduli stabilization mechanisms of the LVS type, new non-trivial constraints arise. In the KKLT scenario, the constrains are milder, but contributions to the μ and $B\mu$ parameters in the Higgs sector can in general not be ignored [2].

1.4 Baryogenesis as a Probe of High-Scale Physics

The discrepancies in the densities of matter and antimatter is one of the most striking features of our universe: while matter make up galaxies, stars and humans, antimatter on the other hand, is nowhere found in large quantities. Furthermore, even the matter content of the universe only make up around 5% of the total energy density of the universe, and understanding the smallness of these numbers is the topic of baryogenesis.

While historically, the baryon asymmetry was conceived of as a cosmological initial condition, it has by now long been understood that the this asymmetry may arise from dynamics satisfying the three Sakharov conditions [36]:

1. The theory must admit baryon number violation.
2. The theory must admit CP-violation, so as to produce a difference in the production rates between particles and antiparticles.
3. The physics must at some point have been out of thermal equilibrium.

A number of physically well-motivated models satisfying these three conditions have been proposed over the years. In *GUT baryogenesis*, the additional gauge bosons of the unified gauge group that do not correspond to gauge bosons of the standard model mediate baryon number violating interactions, and CP-violation arise in these models at loop level. Furthermore, as the temperature drops below the GUT-scale of around 10^{16} GeV, the thermal production of the additional gauge bosons freeze out, and an asymmetry between matter and antimatter may be produced. *Electroweak baryogenesis*, on the other hand, proceed through the interactions of the standard model, which classically preserve baryon number. At the quantum level, tunneling transitions between different vacua with different baryon numbers occur, and for temperatures of the order of the electroweak scale, baryogenesis could proceed. Furthermore, in *leptogenesis*, the baryon asymmetry proceeds through an initial creation of lepton number, which then may decay to a non-vanishing baryon number. Finally, baryogenesis may proceed through the *Affleck-Dine* mechanism, in which a vacuum expectation value of a scalar carrying baryon number decays into the baryons of our universe.

All these models have strengths and weaknesses. In particular, GUT baryogenesis typically requires a reheat temperature of the order of the GUT scale, which can be hard to accommodate in theories of inflation — in particular when considering also other cosmological constraints, e.g. those following from the cosmological gravitino problem. Electroweak baryogenesis does not produce a sufficiently large baryon number, unless specific extensions of the standard model is included in the model. While in neither of these two models are firmly excluded experimentally, they do not appear to readily account for the baryon asymmetry of the universe, and we will now turn to leptogenesis and Affleck-Dine baryogenesis, which both appear to naturally be able to produce a sufficiently large baryon number.

Leptogenesis may be implemented through a number of microscopic models, the simplest involving the 'see-saw' mechanism for neutrino masses, in which each left-handed neutrino of the Standard Model is paired with a right-handed neutrino, which obtains a mass at some high scale, M. The see-saw mechanism can explain the smallness of the Standard Model neutrino masses, and if the right-handed neutrino violates lepton number, then quantum effects can lead to a sufficiently large baryon number. We refer the interested reader to e.g. [37] for a more detailed review of this model. In its simplest implementations, the scale of the mass of the right-handed neutrino must be at least a couple of orders of magnitude below the Planck mass, and it is not obvious that this mechanism gives rise to any strong constrains on string theory models of particle physics.

In Affleck-Dine baryogenesis, baryon number and CP-violation is explicit in the Lagrangian through the structure and phases of the non-renormalizable operators of supersymmetric extensions of the MSSM, and since the mechanism relies on the non-thermal creation of a scalar condensate, the last Sakharov condition is also satisfied. As we will review in §3.1.1, the Affleck-Dine mechanism is particularly interesting from a string theory perspective as it relies on the structure of Planck-mass suppressed operators in the Lagrangian. This structure can be re-phrased as a geometric condition on the field space geometry, and we will show in §3.2.2 how particular models of inflation in string theory are incompatible with Affleck-Dine baryogenesis [3]. Furthermore, we will show that models of Affleck-Dine baryogenesis with non-trivial field dynamics during inflation can be constrained by the high-precision studies of the temperature anisotropies of the CMB, and satellite experiments such as Planck may exclude most models of this type.

1.5 de Sitter Vacua and the String Landscape

In Chapter 4, we will discuss the abundance of metastable vacua with a positive cosmological constant, and we will in particular compute the ratio of the number metastable critical points to the number of saddle points and local maxima [4]. In this section, we will put this analysis in perspective by reviewing a heuristic — yet ultimately flawed — argument for the density of vacua in the string landscape. Consider a theory with N scalar fields, which are assumed to model the light moduli in the effective theory. For large N, computing the potential explicitly for a specified string compactification is prohibitively complicated, but the number of vacua of the theory can be estimated for potentials which are separable into single-field potentials, i.e.

$$V(\phi_1, \ldots, \phi_N) = \sum_{i=1}^{N} V_i(\phi_i)\,. \tag{1.76}$$

Critical points satisfy $\partial_i V = 0$, and if for each single-field potential, the equation $\partial_i V_i = 0$ has a (geometric) average of α solutions, then the total number of critical points is α^N. If the single-field potentials with many critical points, the number of local minima approximately equals the number of local maxima. It follows that the curvature of the potential (or, equivalently, the mass-squared) at each critical point is positive for about half the total number of critical points and negative for the other half. The number of meta-stable vacua is then given by,

$$N_{vac,N} = \alpha^N \left(\frac{1}{2}\right)^N\,, \tag{1.77}$$

which is a rapidly growing number with N for $\alpha > 2$. As an example, for $\alpha = 10$, the number of vacua for systems with less than 2000 fields is,

$$N_{vac,tot} = \sum_{N=1}^{1999} N_{vac,N} \approx 2 \cdot 10^{1397}\,, \tag{1.78}$$

which far exceeds the 10^{120} vacua necessary for solving the cosmological constant by having many solutions.

However, in Chapter 4, we will find that the fraction of meta-stable critical points typically scale like e^{-cN^p} with $p \approx 2$ for sufficiently large N, and this has important consequences. For example, for $c = .3$, and $p = 2$, while still keeping the scaling of critical points exponential in N with $\alpha = 10$, the total number of vacua *for any number of fields* is given by,

$$N_{vac,tot} = \sum_{N=1}^{\infty} N_{vac,N} = \sum_{N=1}^{\infty} \alpha^N e^{-cN^2} \approx 267\,, \tag{1.79}$$

which is far fewer than the 10^{120} required for a solution of the cosmological constant problem based on many solutions. Of course, here the assumption of the scaling of the number of critical point with N was still obtained from the simplistic argument based on single-field potentials, and the full supergravity expression for the density of vacua is more complicated [41], and we will not discuss it here.

The key insight of [4], is that the spectrum of the Hessian matrix can be modeled by a matrix model to a surprisingly high accuracy, and that these model allows for an analytic understanding of the stability properties of random supergravity Hessians. A matrix model for an ensemble matrices denoted M can be defined through a partition function,

$$Z = \int \mathcal{D}M\, e^{-\beta U(M)}\,, \tag{1.80}$$

where the entries of the matrix M are assumed to be *independent* and *identically distributed*, (i.i.d.). The 'free energy' U, provides a weight for the different matrices in the ensemble, and has to be choosen *a priori*. The strength of the topic of random matrix theory is that, at large N the matrices exhibit universality, and under mild restrictions on U, the details of the initial distribution does not matter for the most interesting question. Instead, the matrix models fall into different universality classes that can be distinguished based on their symmetries.

For diagonalizable matrices such as the Hermitian mass matrix of supergravity, equation (1.80) can be expressed after diagonalization of each constituent matrix M of the ensemble, as an integral over the unitary matrices (by the Haar measure), multiplied by an integral over the eigenvalues. This change of coordinates comes with a Jacobian in the form of the *Vandermonde determinant*, which is interpreted as an interaction term between the eigenvalues. In these coordinates, and after suppressing the integration over the unitary group, the partition function is given by,

$$Z = C_N \int_{\Omega} \left(\prod_i^N d\lambda_i \right) e^{-\beta V(\lambda_1,\ldots,\lambda_N)} , \tag{1.81}$$

which can now be interpreted as a statistical system of N particles with potential $V(\lambda_1, \ldots, \lambda_N)$. The domain of the eigenvalues can be taken to be $(-\infty, \infty)$. To compute the probability of a fluctuation to positivity of all eigenvalues of a matrix, the restricted partition function can be found as [39, 40],

$$Z[a] = C_N \int_a^{\infty} \left(\prod_i^N d\lambda_i \right) e^{-\beta V(\lambda_1,\ldots,\lambda_N)} , \tag{1.82}$$

in terms of which,

$$P(\lambda_{min} \geq a) = \frac{Z[a]}{Z[-\infty]} . \tag{1.83}$$

In forthcoming publications [42], we use this formalism to address a number of interesting questions, including an analytical determination of the value of p in the parametrization of the fraction of vacua to critical points as e^{-cN^p}, to 2 for an approximate model of random supergravity discussed in Chapter 4. Extensions of this formalism appears very powerful in addressing a number of interesting physical questions in cosmology and particle physics, but a discussion thereof would go beyond the scope of this thesis. In Chapter 4, we apply the results of [39, 40] directly to obtain an analytical understanding of the high suppression of the fluctuation probability.

With this review, let us now turn to the individual research questions addressed in this thesis, starting with the topics in which Planck-suppressed operators play an important role: sequestering in string compactifications is discussed in Chapter 2, and Affleck-Dine baryogenesis in 3. We will close the thesis by addressing the question of the number of vacua in the string landscape in Chapter 4.

CHAPTER 2

SEQUESTERING IN STRING COMPACTIFICATIONS

2.1 Introduction

The stabilization of the electroweak scale is one of the most significant open questions in theoretical physics. Low-energy supersymmetry provides an elegant solution, but supersymmetry must be broken, and the experimental signatures are principally governed by the supersymmetry-breaking soft terms. In the most plausible scenarios, supersymmetry is broken in a hidden sector and this breaking is mediated to the visible sector by some form of interaction. The structure of the resulting soft terms is largely controlled by the nature of the mediating interaction, motivating efforts to study the mediation of supersymmetry breaking without making reference to the details of the hidden sector.

Naturalness suggests that the visible and hidden sectors should be coupled by non-renormalizable operators induced by integrating out new interactions near the Planck scale. In the celebrated gravity mediation scenario, these couplings provide the leading interaction between the two sectors, and give rise to soft scalar masses of order the gravitino mass $m_{3/2}$. Unfortunately, extremely little is known about Planck-scale interactions, yet some detailed properties of these interactions — at least, as encoded in the structure of the effective theory below the Planck scale — are required in order to make predictions. For instance, strong bounds on flavor violation force the fermion and sfermion mass matrices to be diagonal in the same basis, to high accuracy. It has proved difficult to justify symmetries of a Planck-scale theory that can enforce such a flavor structure. This supersymmetric flavor problem is a serious obstacle to successful phenomenology in high-scale mediation.

The flavor problem in gravity-mediated supersymmetry breaking could be ameliorated if the soft masses in the visible sector were parametrically suppressed compared to $m_{3/2}$. In this case it is possible for scalar mass contributions from some other mediation mechanism, e.g. anomaly mediation, to give rise to visible sector masses with acceptable flavor structure. In such a situation, one says that the source of supersymmetry breaking is *sequestered* from the visible sector [43].

Sequestering amounts to a suppression of the soft terms compared to the natural level induced by 'generic' Planck-suppressed operators coupling the hidden and visible sectors. This state of affairs is unnatural unless it is enforced by a symmetry or other structure in the Planck-scale theory, e.g. extradimensional locality, and the success or failure of sequestering depends very sensitively on Planck-scale interactions. This strongly motivates studying sequestering in string theory, where such contributions can in principle be computed.

Randall and Sundrum originally proposed that sequestering could arise as a result of geometric separation in an internal space [43]: locality in the extra dimensions, where only gravity was assumed to propagate, severely restricted the form of the lower dimensional effective theory. At first sight the extradimensional construction in [43] appears amenable to a realization in string theory, but on closer examination the mechanism of spatial separation does not manifestly extend to string compactifications with moduli: light moduli could easily mediate interactions of gravitational strength, while sequestering requires far feebler interactions. Indeed, Anisimov et al. [44] have argued that for precisely this reason, sequestering is difficult to obtain in certain classes of string compactifications, while Kachru et al. [45] observed that even stabilized moduli are very generally too light to decouple in the manner required.

Nevertheless, it was shown in [46] that sequestering of a large class of operators is natural in certain highly-warped string compactifications: this is the gravity dual of conformal sequestering [47] (see also earlier work in [48]). In the language of the dual approximately-conformal field theory, a contribution to the soft terms of the visible sector fields C mediated by a coupling of the form

$$\int d^4\theta \, C^\dagger C \, O_\Delta \, , \tag{2.1}$$

where O_Δ is an operator of dimension Δ in the CFT, is suppressed by a factor

$$M^2 \sim \left(\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}}\right)^{\Delta-4} m_{3/2}^2 \, , \tag{2.2}$$

with $\Lambda_{\rm IR}, \Lambda_{\rm UV}$ the infrared and ultraviolet scales, respectively, in the CFT. In gravity language, supersymmetry breaking is mediated by perturbations to the supergravity background, and in suitable warped throat solutions – e.g., a Klebanov-Strassler throat attached to a compact space – these perturbations decay rapidly away from the source, sequestering the breaking of supersymmetry.

Crucially, the analysis of [46] was performed in the no-scale limit, i.e. with the complex structure and dilaton stabilized by fluxes, but with the Kähler moduli unstabilized. One should therefore ask whether the sequestering observed in [46] persists upon stabilization of the Kähler moduli. More specifically, the absence of superpotential cross-couplings between the visible and hidden sectors is a requirement for sequestering, as we shall review in more detail in §**??**; by nonrenormalization of the superpotential it is straightforward to arrange that no such coupling arises in perturbation theory. However, such cross-couplings are likely to arise at the nonperturbative level. Nonperturbative superpotentials for the Kähler moduli T can induce new contributions to the soft masses via interactions of the form

$$\Delta W = \mathcal{O}_{vis}\, e^{-aT} \,, \tag{2.3}$$

where a is a constant and $\mathcal{O}_{vis}$ is a gauge-invariant chiral operator composed of visible sector superfields, which to cubic order in the MSSM fields can be written as

$$\mathcal{O}_{vis} = \mu H_u H_d + \lambda^u_{ij} Q^i u^j H_u + \lambda^d_{ij} Q^i d^j H_d + \lambda^l_{ij} L^i e^j H_d \,. \tag{2.4}$$

We have used the standard notation for the chiral superfields of the MSSM, the indices $i, j = 1, 2, 3$ run over families, and μ and $\lambda^{u,d,l}_{ij}$ are constants that are *not* necessarily related to the tree-level μ term and Yukawa matrices, respectively (cf. §**??**).

One might be tempted to ignore nonperturbatively-small interactions, but this is not consistent in vacua for which nonperturbative effects play a critical dynamical role. In particular, when the Kähler moduli are stabilized by a nonperturbative superpotential, one must ask whether this superpotential also spoils sequestering. Answering this question is the primary goal of this chapter, and [2].

In brief, we shall find that in very simple toy models of sequestering in KKLT vacua, the nonperturbative superpotential for the Kähler moduli induces soft B terms of order $m_{3/2}$ in a D3-brane 'visible sector', spoiling sequestering, as expected from the above arguments. In more realistic models, the gauge symmetry of the MSSM partially protects the sfermions from superpotential de-sequestering, and the flavor structure depends on the moduli stabilization scenario. In KKLT vacua with a single volume modulus, the sfermions receive flavor-diagonal masses that are suppressed with respect to the gravitino mass, as well as highly suppressed A-terms. On the other hand, the Higgs and Higgsino masses receive corrections of order $m_{3/2}$.[1] For multiple Kähler moduli, these conclusions remain true as long as the Kähler potential is of the sequestered form. We note that the Higgs sector is very sensitive to the details of the global compactification and is thus in no sense sequestered. We then argue that for certain parameter regimes in the Large Volume Scenario, the corrections to the masses of the sfermions are larger, and can introduce significant flavor violation.

The outline of this Chapter is as follows. In §2.1 we critically review arguments for sequestering in supergravity and in string compactifications. In §2.2 we incorporate nonperturbative stabilization of a Kähler modulus in a simple and explicit string theory toy model. In §2.3 we consider nonperturbative superpotential contributions to the soft masses of a more realistic, MSSM-like, visible sector. In §2.3 we comment on issues that depend on the specific realization of the standard model in string theory, and in §2.3.3 we indicate future directions. We close with conclusions in §2.4. In Appendix A.1 we show that warped sequestering survives the relaxation of a technical assumption made in [46], and in Appendix A.2 we give details of the calculation of soft masses in our explicit example.

2.1.1 Extra-dimensional locality and sequestering

In [43], Randall and Sundrum argued that locality in a higher-dimensional spacetime strongly constrains the soft terms observed in a lower-dimensional

[1] Higgs sector masses of this form were considered in [49].

world. Their observation has three key ingredients. First of all, assuming that only the gravity multiplet propagates in the bulk, higher-dimensional locality restricts the form of the Kähler potential, the superpotential and the gauge kinetic function. In [43], this was demonstrated by considering an off-shell formulation of supergravity in which the field $\Phi = 1 + \theta^2 F_\Phi$ houses some of the auxiliary degrees of freedom for the supergravity multiplet. The relevant portion of the Lagrangian is given by

$$\begin{aligned}\frac{1}{\sqrt{-g}}\,\mathcal{L} = &\int d^4\theta f(C^\dagger, e^{-V}C, X^\dagger, X)\Phi^\dagger\Phi + \int d^2\theta\left(\Phi^3 W(C,X) + \tau(C,X)\mathcal{W}_\alpha{}^2\right) \\ &- \frac{1}{6} f(\tilde{c}^*, \tilde{c}, x^*, x)\,(\mathcal{R} + \ldots)\,.\end{aligned} \tag{2.5}$$

Here the visible sector chiral superfields are collectively denoted by C and the hidden sector fields are denoted by X, with lowest components $\tilde{c}$ and x, respectively. The visible sector vector multiplets are collectively denoted by V, with $\mathcal{W}_\alpha$ the gauge field strength superfields, and τ the corresponding gauge kinetic functions, while $\mathcal{R}$ is the four-dimensional Ricci scalar. The f function is related to the Kähler potential by

$$f = -3M_{Pl}^2\, e^{-K/(3M_{Pl}^2)}. \tag{2.6}$$

The *assumption* that the visible sector communicates with the hidden sector only through the gravity multiplet implies that in the supersymmetric flat space limit, the visible and hidden sectors must decouple. Formally, in this limit $\mathcal{R} = 0$ and $\Phi = 1$, so that one finds

$$f(C^\dagger, e^{-V}C, X^\dagger, X) = f_{\mathrm{hid}}(X^\dagger, X) + f_{\mathrm{vis}}(C^\dagger, e^{-V}C)\,, \tag{2.7}$$

$$W(C,X) = W_{\mathrm{hid}}(X) + W_{\mathrm{vis}}(C)\,, \tag{2.8}$$

$$\tau = \tau_{\mathrm{hid}}(X) + \tau_{\mathrm{vis}}(C)\,. \tag{2.9}$$

We will refer to these conditions collectively as *separability*. The condition (2.7) is equivalent to the statement that the Kähler potential takes the special form

$$K = -3M_{Pl}^2 \ln\left(-\frac{f_{\text{vis}} + f_{\text{hid}}}{3M_{Pl}^2}\right). \tag{2.10}$$

The second observation in [43] concerns the vanishing of the tree-level soft terms. Let us define as usual the $B\mu$ term, the trilinear A terms A_{abc}, and the soft masses $M^2_{a\bar{b}}$ of the MSSM as

$$\mathcal{L}_{soft} = M^2_{a\bar{b}} C^a \bar{C}^{\bar{b}} + \left(\frac{1}{2} B_{ab} C^a C^b + \frac{1}{6} A_{abc} C^a C^b C^c + h.c.\right), \tag{2.11}$$

where the visible sector fields C^a include the Higgses, and in the MSSM $B_{h_u h_d} = B\mu$ is the ordinary $B\mu$ term. Then for superpotentials and Kähler potentials of the form[2] (2.7) - (2.8), one can verify that $A_{abc} = M^2_{a\bar{b}} = 0$.[3] Summarizing, the separable structure (2.7) - (2.8) combined with the absence of supersymmetric visible sector masses leads to sequestering, in that the hidden sector does not induce any soft terms in the visible sector at tree level in supergravity.

When the Planck-scale theory does not respect any flavor symmetry, acceptable flavor structure in the low-energy theory requires suppression of the gravity-mediated soft terms. Sequestering due to extradimensional locality provides a promising mechanism for such suppression, but does not constitute a complete mediation scenario. Instead, sequestering clears the way for small contributions to the soft masses — which would be overwhelmed by gravity-mediated contributions if the latter were present — to dictate the visible sector spectrum. The third key ingredient in [43] was the proposal that anomaly mediation could yield satisfactory soft terms in sequestered configurations. The goal

[2] We will see in §2.1.4 that separability to all orders in the visible sector fields is an unnecessarily strong requirement, and in fact a weaker condition is sufficient to suppress the soft terms.

[3] The separable structure of (2.7) - (2.8), already noticed in [50], does not automatically imply that the $B\mu$ term is small, but does ensure the vanishing of gravity-mediated $B\mu$ terms in the absence of supersymmetric mass terms for the visible sector fields.

of the present Chapter is to investigate the possibility of sequestering in string compactifications, leaving for the future the task of constructing a mediation scenario, and so we will not have more to say about anomaly mediation.

2.1.2 Corrections from moduli-mediated interactions

A key requirement for the argument of [43] is that the extra dimensions must be barren, with only the gravity multiplet propagating in the bulk. The proposal was that any fields not in the gravity multiplet would obtain masses at least as large as the Kaluza-Klein scale,

$$m \gtrsim \frac{1}{R}\,, \tag{2.12}$$

with R a typical length scale of the compactification. This would lead to an e^{-mR} Yukawa suppression of the real-space propagator of these fields, which in turn would give rise to a large suppression of any effect these fields might have on the soft terms of a visible sector separated from the hidden sector by a distance R.

In practice, barren extra dimensions are quite rare, both in string compactifications and in more general extradimensional model-building: compactification moduli typically induce new gravitational-strength interactions that mediate supersymmetry breaking and hence spoil sequestering. The authors of [44] examined a variety of string theory models with calculable Kähler potentials, and found that the special form (2.10) does not seem to be generic in M-theory or string theory with branes, despite the manifest extradimensional locality of such models. The unwanted couplings arise from the exchange of bulk supergravity fields, particularly moduli.

Along the same lines, the authors of [45] gave a general argument showing that the assumption of barren extra dimensions does not hold in string compactifications, even after stabilization of the moduli. In any compactification for which moduli stabilization can be described in the four-dimensional effective theory, the moduli masses will be no larger than the cutoff scale of the four-dimensional effective theory, and in particular will not exceed the Kaluza-Klein scale, so that $mR \ll 1$. Thus, the effects transmitted by massive, stabilized moduli cannot be neglected in general.

The importance of moduli stabilization for sequestering was first emphasized by Luty and Sundrum in [51]. They considered five-dimensional supergravity compactified on $S^1/\mathbb{Z}_2$ and asked whether supersymmetry breaking on the hidden orbifold boundary gave rise to sequestered supersymmetry breaking for matter fields on the visible brane. Prior to stabilization of the radion controlling the interval size, the Kähler potential took a sequestered form. To stabilize the radion, they invoked gaugino condensation in a bulk gauge group and in a boundary gauge group, yielding a superpotential

$$W = c + b\,e^{-aT}\,, \tag{2.13}$$

for constants a, b, c, and with T the radion. Luty and Sundrum then showed that with this superpotential, sequestering survives the stabilization of the radion.

In string theory, the situation is somewhat more complicated, for several reasons. To assist the reader in navigating the remainder, we briefly sketch these complications. First, even before stabilization of the moduli, the Kähler potential for an unwarped string compactification does *not* generically take the sequestered form (2.10), as we have just reviewed. However, strong warping ameliorates some of the moduli-mediated interactions, as we shall explain in §2.1.3.

Moreover, a criterion weaker than (2.7) for the suppression of Kähler potential couplings appears well-motivated in certain unwarped examples, and may allow effective sequestering even in compactifications violating (2.7) (§2.1.4). Most importantly, nonperturbative stabilization of the Kähler moduli introduces new interactions that violate (2.8) and spoil sequestering, even in the presence of warping, as we will explain in §2.1.5.

2.1.3 Sequestering in warped compactifications

Although moduli-mediated interactions render sequestering non-generic in unwarped string compactifications [44], some of the problematic effects are suppressed by strong warping [46]. Suppose that the supersymmetry-breaking sector is localized at the bottom of a warped throat. From the ten-dimensional perspective, the mediation of supersymmetry breaking to a visible sector some distance up (or even outside) the throat will proceed through perturbations of the supergravity fields sourced in the infrared, i.e. from the bottom of the throat. Taking the throat to be a warped Calabi-Yau cone with Sasaki-Einstein base X_5, the bulk fields φ can be expanded in eigenmodes on X_5, so that schematically one has

$$\varphi = \sum_{\alpha} c_\alpha r^{-\Delta_\alpha} Y_\alpha(\Psi). \tag{2.14}$$

Here α indexes the quantum numbers under the isometries of X_5, c_α are constants, r is the radial coordinate, Ψ denotes the angular coordinates on X_5, Y_α is an angular harmonic on X_5, and Δ_α is the dimension of the operator that is dual, via AdS/CFT, to the corresponding supergravity mode. Couplings between a supersymmetry-breaking sector located at the tip of the throat and a visible sector located at the top of the throat are suppressed by powers of the hierarchy of scales in the throat. When all operators inducing cross-couplings have $\Delta > 4$, the gravity-mediated soft terms are highly suppressed, and the system experiences *warped sequestering*, which is the gravity dual of conformal sequestering [47].

Warped sequestering in the Klebanov-Strassler throat

Let us briefly review and clarify the results of [46], which analyzed the mediation of supersymmetry breaking from an anti-D3-brane to a D3-brane by normalizable profiles of the supergravity fields in a Klebanov-Strassler throat region of a no-scale compactification. The leading effects arose from the lightest Kaluza-Klein modes on $T^{1,1}$ — and correspondingly the lowest-dimension operators in the Klebanov-Witten theory [52] — with two properties: the mode must be sourced by an anti-D3-brane, and it must induce supersymmetry-breaking masses for the D3-brane fields. A particular supergravity field, denoted by Φ_- in the conventions of [53] (their equation (2.5)), controls D3-brane scalar masses, and so the task was to determine the lowest modes in the spectrum of Φ_- excitations that are sourced by an anti-D3-brane.[4]

In [46] (to which we refer for explanation of our notation) it was argued, following [56], that the lowest-dimension operator dual to a normalizable mode of Φ_- is a non-chiral operator $\mathcal{O}_{\sqrt{28}}$ with quantum numbers $(\mathbf{3}, \mathbf{3}, \mathbf{1})$ under the $SU(2) \times SU(2) \times U(1)$ global symmetry, and with dimension $\Delta = \sqrt{28} \approx 5.29$. However, upon comparing to the result of [57] for the Coulomb potential between a D3-brane and an anti-D3-brane in a warped throat, the authors of

[4]We implicitly assume that the anti-D3-brane configuration corresponds to a supersymmetry-breaking state of the cascading gauge theory, as in [54], but it would be valuable to confirm or exclude this along the lines of [55].

[46] found that this mode is apparently not sourced by an anti-D3-brane, and the leading mode of Φ_- in the actual solution is dual to the non-chiral singlet $\mathcal{O}_8 = \int d^4\theta \, \mathrm{tr}\left(\mathcal{W}_\alpha \mathcal{W}^\alpha \overline{\mathcal{W}}_{\dot\beta} \overline{\mathcal{W}}^{\dot\beta}\right)$ with dimension $\Delta = 8$. This result, which was subsequently confirmed in [58, 59], might lead one to expect that there exists a different supersymmetry-breaking state (potentially preserving different global symmetries) in which the apparently more relevant operator $\mathcal{O}_{\sqrt{28}}$ obtains an expectation value and leads to less-sequestered soft terms.

This expectation would be erroneous: although the operator $\mathcal{O}_{\sqrt{28}}$ is indeed present in the Klebanov-Witten theory, the vev of $\mathcal{O}_{\sqrt{28}}$ is *not* dual to a normalizable perturbation of Φ_-, and moreover the lowest-dimension operator whose expectation value is dual to a normalizable perturbation of Φ_- is precisely the operator $\mathcal{O}_8$ induced by an anti-D3-brane. Therefore, the anti-D3-brane activates the most relevant Φ_- perturbation available in the theory, and moreover this mode is a singlet, as one would expect at leading order in a multipole expansion.

To correct the assignments of operators to supergravity modes, we refer to the discussion in §3.3 of [53]. There it was observed that a mode of Φ_- dual to the vev of an operator with dimension Δ has a radial profile $\delta\Phi_- \propto r^{4-\Delta}$, which differs by a factor r^4 from the result for a scalar field in AdS_5 with the standard normalization. As a result, a given operator in the Klebanov-Witten theory whose expectation value is dual to a normalizable mode of Φ_- will have its source dual to a non-normalizable mode of an independent supergravity field [53], which we denote Φ_+. In particular, as explained in [53], the simplest operators dual to normalizable Φ_- profiles are of the form $\mathrm{Tr}\left(\mathcal{W}_\alpha \mathcal{W}^\alpha \overline{\mathcal{W}}_{\dot\beta} \overline{\mathcal{W}}^{\dot\beta} (AB)^k\right)$, with k a non-negative integer. The lowest operator in this tower is the $\Delta = 8$ singlet described above.

What of the operator $\mathcal{O}_{\sqrt{28}}$? Its expectation value is dual to a normalizable mode of Φ_+ (while a source for this operator is dual to a non-normalizable mode of Φ_-). Therefore, a vev of this operator does not induce soft terms for a D3-brane visible sector in the ultraviolet.

However, there is a mode with much lower dimension that could potentially induce soft terms at the nonlinear level: this is a mode of flux dual to the chiral operator $\mathcal{O}_{5/2} = \int d^2\theta \, \mathrm{tr}(A^i B^j)$ with quantum numbers $(\mathbf{\frac{1}{2}}, \mathbf{\frac{1}{2}}, -\mathbf{1})$ and dimension $\Delta = 5/2$ (cf. [60]). In Appendix A.1 we demonstrate that this mode does not alter the conclusions of [46].

In conclusion, for anti-D3-brane supersymmetry breaking in the Klebanov-Strassler solution, the lowest-dimension operator mediating soft terms to a D3-brane 'visible sector' has $\Delta = 8$, so that the sequestering is very strong.

The considerations described above are directly applicable only to a non-compact Klebanov-Strassler throat. For a finite throat region attached to a compact space, there is at least one new light degree of freedom, the Kähler modulus T controlling the overall volume. Because T is not part of the CFT, it is natural to expect that T will mediate soft terms that are *not* suppressed by the hierarchy of energy scales in the throat. In §2.1.5 we will observe that nonperturbative superpotentials for the Kähler moduli indeed generically spoil warped sequestering. However, we first describe a weaker criterion for sequestering which is applicable in certain cases with a well-localized visible sector.

2.1.4 Sort-of sequestering

We have reviewed how extradimensional locality and the assumption of barren extra dimensions imply the separability of W and f, as in (2.7) and (2.8), and how (2.7) and (2.8) in turn imply sequestering of supersymmetry breaking. Because light moduli in string compactifications violate (2.7), it is reasonable to ask whether a weaker assumption might suffice to ensure suppression of the soft masses in comparison to $m_{3/2}$.

To identify this weaker condition, we expand a general Kähler potential and superpotential in powers of the visible sector fields C^a as

$$K = \widehat{K}(X,\bar{X}) + \tilde{K}_{a\bar{b}}(X,\bar{X})C^a\bar{C}^{\bar{b}} + \left[Z(X,\bar{X})H_uH_d + h.c.\right]\dots \tag{2.15}$$

$$W = \widehat{W}(X) + \mu(X)H_uH_d + \frac{1}{6}Y_{abc}(X)C^aC^bC^c + \dots \tag{2.16}$$

and then require the separability of the f function *only at leading order*[5] in the visible sector fields [61]. This condition, which is related to the "extended no-scale structure" in [61], reads

$$\tilde{K}_{a\bar{b}} = e^{\widehat{K}/3M_{Pl}^2}\kappa_{a\bar{b}}\,, \tag{2.17}$$

with $F^m\partial_m\kappa_{a\bar{b}} = 0$, i.e. $\kappa_{a\bar{b}}$ does not depend on the moduli that get non-vanishing F-term vevs. In particular this means that one can always rotate and rescale[6] the C^a such that $\kappa_{a\bar{b}} \to \delta_{a\bar{b}}$. Using (2.17) and the standard supergravity formulae [102, 63, 64] it is easy to verify that a series of cancellations leads to the following result for the soft terms of the MSSM fields (not yet canonically normalized,

[5]In terms of the notation of §2.1 this means that f can have a part $f_{\rm mix}$ involving both the hidden sector fields and the visible sector fields as long as it satisfies $\partial_a f_{\rm mix}(X,\bar{X},C,\bar{C}) = \partial_a\bar{\partial}_{\bar{b}} f_{\rm mix}(X,\bar{X},C,\bar{C}) = 0$.

[6]For a generic moduli dependence, it is possible to diagonalize $\tilde{K}_{a\bar{b}}(X,\bar{X})$ only at a single point in the moduli space. As computing the soft terms (2.61)-(3.84) requires differentiating with respect to the moduli, this is not sufficient.

which we emphasize with the hats):

$$
\begin{aligned}
\hat{M}^2_{a\bar{b}} &= \frac{2}{3}\frac{V_0}{M_{Pl}^2}\tilde{K}_{a\bar{b}} \simeq 0\,, && (2.18)\\
\hat{A}_{abc} &= e^{\widehat{K}/2M_{Pl}^2} F^m \partial_m Y_{abc}\,, && (2.19)\\
B\hat{\mu} &= e^{\widehat{K}/2M_{Pl}^2}\mu\left[F^m\left(\partial_m \log\mu - \frac{\widehat{W}_m}{3\widehat{W}}\right)\right] + \mu\frac{V_0}{3\widehat{W}} + O(Z)\,, && (2.20)
\end{aligned}
$$

where V_0 is the vacuum energy at the minimum of the F-term potential, which we are assuming is negligibly small, and $O(Z)$ stands for terms proportional to Z and its derivatives, which we omit for simplicity. It is clear from (2.19) that in order to ensure the absence of gravity/moduli mediated A terms[7] one needs to assume the separability (2.8) of the superpotential, such that $F^m \partial_m Y_{abc} = 0$.

As for the all-orders separability in (2.7) and (2.8), the leading-order separability condition (2.17) is also not generically satisfied in string compactifications. Nevertheless, there are arguments that (2.17) might be valid, at least approximately. In fact, in [65] it was argued that the combination of locality and holomorphicity enforces a special form of the metric on the visible sector moduli space. Their argument, which we will review in §2.3.2, suggests that $K_{a\bar{b}} \sim e^{K/3M_{Pl}^2}\kappa_{a\bar{b}}$ with $\kappa_{a\bar{b}}$ independent of the Kähler moduli. When all other moduli have only small or vanishing F-terms then interesting suppressions of the soft terms as in (2.18) and (2.19) might arise.

2.1.5 Superpotential de-sequestering

Even when the separability (2.7) of f can be justified in a scenario with barren extra dimensions, or when the weaker criterion (2.17) follows from locality and

[7]For $B\mu$ even the separability of both f and W is not sufficient to ensure sequestering. Instead one needs to make further assumptions, e.g. the absence of supersymmetric visible sector masses, i.e. $\mu = Z = 0$.

holomorphicity, the separability (2.8) of W is necessary to prevent flavor violation in the soft trilinear A terms.

We now make a critical observation: the separability of W, (2.8), is generically violated in string compactifications stabilized by nonperturbative effects, and the resulting soft terms therefore require careful study. This is one of the main goals of the present Chapter.

The best-understood scenarios for complete moduli stabilization in type IIB string theory [26, 27, 66] incorporate nonperturbative contributions to the superpotential, e.g. from gaugino condensation on a stack of D7-branes wrapping a four-cycle, to lift the Kähler moduli. Consider a visible sector residing on (possibly fractional) D-branes in a compactification of type IIB string theory. Suppose that a Kähler modulus T describing the volume of some four-cycle Σ is stabilized by gaugino condensation in a super Yang-Mills sector on N_c D7-branes wrapping Σ. Even if Σ is distant from the D-branes constituting the visible sector, strings stretching between the hidden and visible D-branes carry charges under both sectors. Integrating out these strings will generically induce couplings between the sectors. This computation has been performed explicitly in toroidal orientifolds (i.e. without warping), with the result that D-branes distant from the hidden sector give unsuppressed threshold corrections to the nonperturbative superpotential [67]. An important question is whether these stretched strings can be massive enough to decouple if the hidden and visible sectors are well-separated along a warped direction. A precisely analogous question arises in D3-brane inflation, in which significant contributions to the inflaton potential arise from strings stretched between the inflationary D3-brane and the D7-branes whose strong gauge dynamics stabilizes the Kähler moduli.

Explicit computation of the nonperturbative superpotential has revealed that the induced cross-couplings are *not* negligible, even in strongly warped backgrounds [68]. The physical explanation for this was provided in [69], which showed that in any warped throat with Sasaki-Einstein base, the mass of a string stretching up the throat is small compared to the four-dimensional Planck mass.

We therefore expect that nonperturbative stabilization of the Kähler moduli can induce new contributions to the soft masses via superpotential interactions[8] as in equation (2.3), $\Delta W = \mathcal{O}_{vis}\, e^{-aT}$.

Nonperturbative stabilization of the radion was also considered in [51], and was found to be compatible with sequestering. The critical difference between [51] and the present work is that we allow couplings to the visible sector in the nonperturbative superpotential for the Kähler moduli, so that b in (2.13) would be a gauge-invariant combination of visible sector fields, cf. (2.4), rather than a pure constant. Clearly, this dramatically changes the physical outcome. Let us stress that the superpotential (2.13) of [51] does follow upon assuming the absence of couplings between the hidden and visible sectors. However, *in string compactifications for which the hidden and visible sectors are composed of D-branes*, one invariably has a spectrum of massive strings stretching between these D-branes, and integrating out these strings induces cross-couplings of the form (2.3).

[8]We remark that perturbative nonrenormalization cannot forbid nonperturbative couplings of this form. Moreover, as long as T is not charged under the symmetries of the Standard Model, these symmetries also cannot forbid the couplings (2.3).

2.2 The Effects of Moduli Stabilization: A Toy Model

We have argued above that in a compactification whose Kähler moduli are stabilized by a nonperturbative superpotential, superpotential cross-couplings (2.3) between the visible sector and the Kähler moduli induce soft supersymmetry breaking in the visible sector. To assess the form of the resulting soft terms, we turn to a string theory toy model with stabilized moduli in which the resulting soft terms can be computed explicitly.

2.2.1 Supersymmetric vacuum for a D3-brane

As in [46], we consider a D3-brane in a Klebanov-Strassler throat [70]. The D3-brane will serve as a proxy for the visible sector, not because its low-energy effective theory gives a good approximation to the phenomenological features of the standard model, but because it is a simple but nontrivial case where one can test the warped sequestering proposal. First, in §2.2.1, we recall the essentials of KKLT moduli stabilization [26] and then, in §2.2.1, we obtain a supersymmetric vacuum[9] for a D3-brane in the conifold. The soft terms induced by supersymmetry breaking are then obtained in §2.2.2 and evaluated in §2.2.3. In §2.2.4 we extract a few lessons from the toy model.

[9]The solution that we will investigate in detail was first noticed in [71], but we mention in §A.2.2 how this extends to a broader class of solutions.

The KKLT scenario

In warped compactifications of type IIB string theory on conformally Calabi-Yau three-folds with flux and O-planes, the four-dimensional effective theory contains complex structure moduli, Kähler moduli, and the axio-dilaton, as well as open string moduli due to branes. For simplicity, we will consider the case of a single Kähler modulus T, and a single spacetime-filling D3-brane whose position is parameterized by three complex scalars ϕ_i, $i = 2, 3, 4$. Three-form flux generates a classical superpotential for the axio-dilaton and complex structure moduli,

$$W_0 = \int G_3 \wedge \Omega, \tag{2.21}$$

providing these moduli with reasonably large masses. After integrating out the massive moduli[10] and introducing a nonperturbative superpotential from gaugino condensation on n D7-branes, the $\mathcal{N} = 1$ effective theory for the Kähler modulus and D3-brane matter fields takes the form:

$$K = -3M_{Pl}^2 \ln\Big(T + \bar{T} - \gamma k(\phi, \bar{\phi})\Big) \equiv -3 \ln U, \tag{2.22}$$

$$W = W_0 + W_{\rm np} = W_0 + \mathcal{A}(\phi) e^{-aT}\,, \tag{2.23}$$

where $k(\phi, \bar{\phi})$ is the Kähler metric on the Calabi-Yau manifold, $\gamma = \frac{1}{3M_{Pl}^2}$, $U = T + \bar{T} - \gamma k(\phi, \bar{\phi}) = \mathcal{V}^{2/3}$, and $a = \frac{2\pi}{n}$.

Before delving into the particulars of explicit models, we readily observe that the nonperturbative effect responsible for the stabilization of the D3-brane and the Kähler modulus involves a direct cross-coupling between these two sectors,

[10]The Kähler potential including the complex structure moduli and dilaton is not of the form (2.10), and thus one might be tempted to conclude that these models do not sequester. However, in the case that the complex structure moduli and axio-dilaton do not obtain large F-terms after supersymmetry breaking [72, 73], these moduli are not part of the supersymmetry-breaking hidden sector, and the condition (2.10) need not apply to these moduli for sequestering to work.

as in equation (2.23). If the D3-brane is regarded as a toy visible sector, a large F-term for T would indeed violate the separability condition (2.8) for sequestering. We now turn to quantifying this contribution to the D3-brane soft mass.

Supersymmetric D3-branes in the conifold

To incorporate moduli stabilization explicitly, we follow [68, 74] and embed a stack of n D7-branes in the throat region, along a divisor $z_2 = \mu$ [75]. Here μ is a complex constant that encodes the D7-brane location, and we are using the standard coordinates in which the deformed conifold is defined by the locus

$$\sum_{A=1}^{4} z_A^2 = \epsilon^2 \tag{2.24}$$

in $\mathbb{C}^4$. Gaugino condensation on the D7-branes then yields the nonperturbative superpotential [68, 76]

$$W_{\mathrm{np}} = \mathcal{A}_0\Big(\frac{z_2 - \mu}{\mu}\Big)^{1/n} e^{-aT}\,, \tag{2.25}$$

where $\mathcal{A}_0$ is a constant with dimensions of $(\mathrm{mass})^3$. Before incorporating the effects of supersymmetry breaking, we obtain a supersymmetric AdS solution by solving the F-flatness equations for the ansatz (2.22), (2.23):

$$D_T W = 0\,, \tag{2.26}$$

$$D_i W = 0\,. \tag{2.27}$$

Using (2.26), the F-flatness conditions (2.27) for the three independent D3-brane coordinates z_i, $i = 2, 3, 4$, can be written as

$$\partial_i\left[\ln\left(\frac{z_2 - \mu}{\mu}\right)\right] - an\gamma k_i = 0\,, \tag{2.28}$$

which, we observe, does not depend on T. This makes it convenient to first find the position of the D3-brane, and then feed this information into (2.26). Each solution has a two real-dimensional moduli space, consistent with the residual $SO(2)$ symmetry of the solution.

We now expand in $|\frac{z_2}{\mu}| \equiv \frac{1}{B}$, keeping only the leading term, to obtain a solution in which the D3-brane is at much smaller radial position than the D7-brane, but is still far above the tip. As we will demonstrate in §2.2.3, this leads to the hierarchy of scales

$$0 \approx \left|\frac{\epsilon}{\mu}\right| \ll \left|\frac{z_2}{\mu}\right| \ll 1, \tag{2.29}$$

and we can consistently set $\epsilon = 0$. We choose to study a D3-brane localized at a specific point in this moduli space, which to this order is defined by $z_3 = z_4 = 0$ and

$$z_2 = \frac{|\mu|}{B} = \frac{1}{4\left(\frac{4\pi\gamma}{3}\right)^3 |\mu|^3}, \tag{2.30}$$

where we have taken z_2 real. In this class of solutions we have $r^3 = 2|z_2|^2$, and $z_1 = \pm i z_2$ is purely imaginary. In our discussion of the masses we will consider the upper sign. Once the location of the D3-brane is found, (2.26) becomes a single-variable transcendental equation, which can be solved numerically. We now turn to the effects of supersymmetry breaking on this vacuum.

2.2.2 Soft masses for the D3-brane

We will break supersymmetry, as in KKLT, by adding an anti-D3-brane at the tip of the throat, which contributes the 'uplift' potential

$$V_{\mathrm{up}} = \frac{D}{(T + \bar{T} - \gamma k(z, \bar{z}))^2}, \tag{2.31}$$

where the constant D is determined by requiring the approximate cancellation of the cosmological constant. The results of [46] ensure that higher-order terms in the expansion of the brane-antibrane potential are negligible.

The supersymmetry-breaking contribution (2.31) to the potential for T and ϕ_i will in general induce shifts in the vevs. Let us use Z^M to denote the vevs of $T, \bar{T}, z_i, \bar{z}_i$, with Z_0^M denoting the vevs in the supersymmetric solution, and $Z_\star^M \equiv Z_0^M + \delta Z^M$ denoting the vevs in the supersymmetry-breaking solution. We will be interested in configurations for which $\delta Z^M \ll Z_0^M$, i.e. the shifts induced by the uplifting are small. In this approximation, we can obtain the shifts δZ^M by solving

$$0 = \partial_M (V_F + V_{up})\Big|_{Z_\star} \approx \partial_M V_{up}\Big|_{Z_0} + \delta Z^N \left[\nabla_N \nabla_M (V_F + V_{up})\right]\Big|_{Z_0} . \tag{2.32}$$

This amounts to inverting the total mass matrix evaluated at the supersymmetric point, $\delta Z^M = -\left(\partial_N \partial_M (V_F + V_{up})\right)^{-1}\Big|_{Z_0} \partial_N V_{up}\Big|_{Z_0}$. One finds (see §A.2.4 for details, including justification for dropping covariant derivatives) that the volume modulus shifts to a slightly larger value. However, the D3-brane remains at the point where it was supersymmetrically stabilized, to leading order in $1/B$ and to quadratic order in $\frac{1}{aU}$, despite the fact that $V_{\rm up}$ depends on the D3-brane coordinates.

We can now obtain the mass matrix for the supersymmetry-breaking vacuum in a Taylor expansion around the supersymmetric point: at leading order we find

$$\nabla_M \nabla_N (V_{\rm tot})\big|_{Z_\star} = \partial^2_{MN} \left(V_{\rm F} + V_{\rm up}\right)\Big|_{Z_\star} \tag{2.33}$$

$$= \partial^2_{MN} \left(V_{\rm F} + V_{\rm up}\right)\Big|_{Z_0} + \delta Z^P \, \nabla_P \partial^2_{MN} \left(V_{\rm F} + V_{\rm up}\right)\Big|_{Z_0}. \tag{2.34}$$

In fact, the contribution proportional to δZ^T turns out to be negligible in this example: explicit evaluation of

$$(\Delta M^2_{\rm tot})_{MN} = \delta Z^P\, \nabla_P \partial^2_{MN} \left(V_{\rm F} + V_{\rm up}\right)\Big|_{Z_0} \tag{2.35}$$

shows that these contributions are suppressed by at least one power of $1/(aU)$ or $1/B$ with respect to the zeroth-order contribution in the expansion of the mass matrix. Thus, to lowest order in $1/B$, $1/(aU)$, the mass matrix is given by the full mass matrix evaluated at the supersymmetric point,

$$\partial^2_{MN}\,(V_{\rm tot})\big|_{Z_\star} \approx \partial^2_{MN}\,(V_{\rm tot})\big|_{Z_0}\,. \tag{2.36}$$

We refer the interested reader to §A.2.4 for the full details.

Now, the scalar masses at the supersymmetric point ($D_A W = 0$) are easily obtained by differentiating the F-term potential (A.9) twice,

$$\partial_a \partial_{\bar b} V_F|_{Z_0} = e^{K/M_{Pl}^2}\left(K^{A\bar B}\partial_a(D_A W)\partial_{\bar b}(\bar D_{\bar B}\overline{W}) - 2\frac{|W|^2}{M_{Pl}^4}K_{a\bar b}\right), \tag{2.37}$$

$$\partial_a \partial_b V_F|_{Z_0} = -\frac{e^{K/M_{Pl}^2}\overline{W}}{M_{Pl}^2}\partial_a D_b W. \tag{2.38}$$

The holomorphic masses in (2.38) would not be present in unbroken rigid supersymmetry, where holomorphic masses only appear in the form of soft B terms. In local supersymmetry, however, these terms can be nonzero without breaking supersymmetry, as discussed in e.g. [9]. The ordinary masses in equation (2.37) are better understood when considered in conjunction with the fermion masses evaluated at the supersymmetric point,

$$m_{ab}\Big|_{Z_0} = e^{\frac{K}{2M_{Pl}^2}}\partial_a D_b W\,, \tag{2.39}$$

$$m^2_{3/2}|_{Z_0} = e^{K/M_{Pl}^2}\frac{|W|^2}{M_{Pl}^4}, \tag{2.40}$$

so, expressed in terms of fermion masses evaluated at the supersymmetric point, we have

$$\partial_a \partial_{\bar b} V_F|_{Z_0} \quad = K^{c\bar d} m_{ac} m_{\bar b \bar d} - 2m^2_{3/2}K_{a\bar b}\,. \tag{2.41}$$

The uplift potential (2.31) induces supersymmetry-breaking masses of the form

$$\partial^2_{MN} V_{up} = \frac{2V_{\rm up}}{3M_{Pl}^2}\left(K_{MN} + \frac{2}{3M_{Pl}^2}K_M K_N\right), \tag{2.42}$$

where M, N can be either holomorphic or antiholomorphic. Using that $V_{\rm tot}|_{Z_0} = (V_F + V_{\rm up})|_{Z_0} \simeq 0$, we can write

$$\partial^2_{a\bar{b}} V_{\rm tot}|_{Z_0} = K^{c\bar{d}} m_{ac} m_{\bar{b}\bar{d}} + \frac{4}{3}\frac{m_{3/2}^2}{M_{Pl}^2} K_a K_{\bar{b}}\,, \tag{2.43}$$

$$\partial^2_{ab} V_{\rm tot}|_{Z_0} = m_{3/2}^2\left[K_{ab} + \frac{4}{3M_{Pl}^2}K_a K_b - \frac{M_{Pl}^2 W_{ab}}{W}\right]. \tag{2.44}$$

The fermion mass-squared now appears in the expression for the nonholomorphic scalar mass-squared, which will be useful when we consider mass splittings.

2.2.3 Evaluated soft masses

In §A.2.4, we express our results for the mass matrix in terms of V_F, aU, and B. While the first two of these quantities are determined completely once $\mathcal{A}_0, W_0$ and $a = \frac{2\pi}{n}$ are specified, the latter is a parameter of the solution describing the position of the D7-brane in the throat. In this section, we explicitly evaluate the mass matrices for a particular set of values for $W_0, \mathcal{A}_0, n$ and B. We caution the reader not to attach undue weight to the precise numbers presented here, which serve only to allow comparison of various contributions to the soft masses.

We will consider the case $|\mathcal{A}_0| = M_{Pl}^3$, $|W_0| = 10^{-13}M_{Pl}^3$, and $a = \frac{2\pi}{32}$, for which $aU \approx 66$. The gravitino mass is then given, to lowest order in $1/(aU)$ and in $1/B$, by

$$m_{3/2}^2\Big|_{Z_*} \simeq \left|\frac{W_0}{M_{Pl}^2}\right|^2 e^{K/M_{Pl}^2}\Big|_{Z_0} \approx 1.3\cdot 10^{-31} M_{Pl}^2 \approx (867\,{\rm GeV})^2\,. \tag{2.45}$$

We now specialize to the case[11] $B = 400$, still using $aU \approx 66$ as above, so that working to lowest order in an expansion in these two quantities provides sufficient accuracy.

The eigenvalues of the full scalar mass-squared matrix at the previously supersymmetric point are $(aU)^2 m_{3/2}^2 \approx 4400\ m_{3/2}^2$ with multiplicity two, corresponding to the real and imaginary parts of the Kähler modulus. There are two flat directions, which in this case are in the imaginary z_3, z_4 directions. The corresponding real directions both have masses $2m_{3/2}^2$. The last eigenvectors are almost aligned with the real and imaginary parts of the z_2 direction. While the imaginary direction has a mass of $\frac{3}{4}m_{3/2}^2$, the real part is tachyonic with a mass-squared of $-\frac{1}{4}m_{3/2}^2$. Several tachyons with mass-squared above the Breitenlohner-Freedman bound were present in the AdS vacuum, but most of these obtained nonnegative masses after uplifting. It is possible that this last tachyon is cured in regions of parameter space that are not accessible to our perturbative expansion in $1/(aU)$ and $1/B$, but as our primary goal is to understand the transmission of soft masses rather than to construct a fully realistic model, we will not consider this point further.

The masses of these real fields will be interpreted in part as supersymmetric masses in Minkowski space, and in part as soft masses. We identify the soft part of the ordinary, non-holomorphic, masses as the mass splittings between bosons and fermions in Minkowski space, while any holomorphic mass term is regarded as a soft B term mass. Evaluating the normalized soft masses, we

[11] See §A.2.5 for consideration of microphysical upper bounds on B.

obtain

$$M^2_{a\bar{b}}\Big|_{Z_*} = \partial^2_{a\bar{b}}(V_F + V_{\text{up}})\Big|_{Z_0} - m_{ab}K^{b\bar{c}}m_{\bar{c}\bar{b}}\Big|_{Z_0} = \frac{4m^2_{3/2}}{3M^2_{Pl}}K_a K_{\bar{b}}\Big|_{Z_0} \tag{2.46}$$

$$= 4m^2_{3/2}\left(\delta^{\rho\bar{\rho}}_{a\bar{b}} - \frac{\sqrt{3}}{2(\pi UB)^{1/2}}(\delta^{\rho\bar{2}}_{a\bar{b}} + \delta^{2\bar{\rho}}_{a\bar{b}}) + \frac{3}{4\pi UB}\delta^{2\bar{2}}_{a\bar{b}}\right) \tag{2.47}$$

$$= 4m^2_{3/2}\begin{pmatrix} 1 & -1.3\cdot 10^{-3} & & \\ -1.3\cdot 10^{-3} & 1.8\cdot 10^{-6} & & \\ & & 0 & \\ & & & 0 \end{pmatrix}. \tag{2.48}$$

The normalized soft B terms are evaluated to

$$B_{ab} \equiv \partial_{ab}V_{\text{tot}}\Big|_{Z_*} = m^2_{3/2}\left[K_{ab} + \frac{4}{3M^2_{Pl}}K_a K_b - \frac{M^2_{Pl}W_{ab}}{W}\right] \tag{2.49}$$

$$= m^2_{3/2}\left(aU\delta^{\rho\rho}_{ab} - \frac{\sqrt{3}a}{2}\left(\frac{U}{\pi B}\right)^{1/2}\delta^{\rho 2}_{\{ab\}} - \frac{1}{2}\delta^{22}_{ab} + \delta^{44}_{ab} + \delta^{33}_{ab}\right) \tag{2.50}$$

$$= m^2_{3/2}\begin{pmatrix} 66 & -8.8\cdot 10^{-2} & & \\ -8.8\cdot 10^{-2} & -\frac{1}{2} & & \\ & & 1 & \\ & & & 1 \end{pmatrix}. \tag{2.51}$$

For completeness, we also list the F-term vevs at the non-supersymmetric solution to the lowest non-vanishing order in the perturbative expansion,

$$F_a|_{Z_\star} = D_a W|_{Z_\star} = D_a W|_{Z_0} + \delta Z^M \nabla_M D_a W|_{Z_0} \tag{2.52}$$

$$= \delta X^T\left(\partial_T D_a W + \frac{K_{\bar{T}a}}{M^2_{Pl}}W\right)\Bigg|_{Z_0}. \tag{2.53}$$

For the canonically normalized volume modulus, we obtain $|F_T|^{1/2} \approx \left(\frac{\sqrt{3U}}{a}m_{3/2}M_{Pl}\right)^{1/2} \approx 5.8\cdot 10^{11}\,\text{GeV}$. In the visible sector, the canonically normalized field corresponding to z_2 gets an F-term with $|F_2|^{1/2} \approx \left(m_{3/2}M_{Pl}\frac{3}{2a}\frac{1}{\sqrt{\pi B}}\right)^{1/2} \approx 2.1\cdot 10^{10}\,\text{GeV}$.

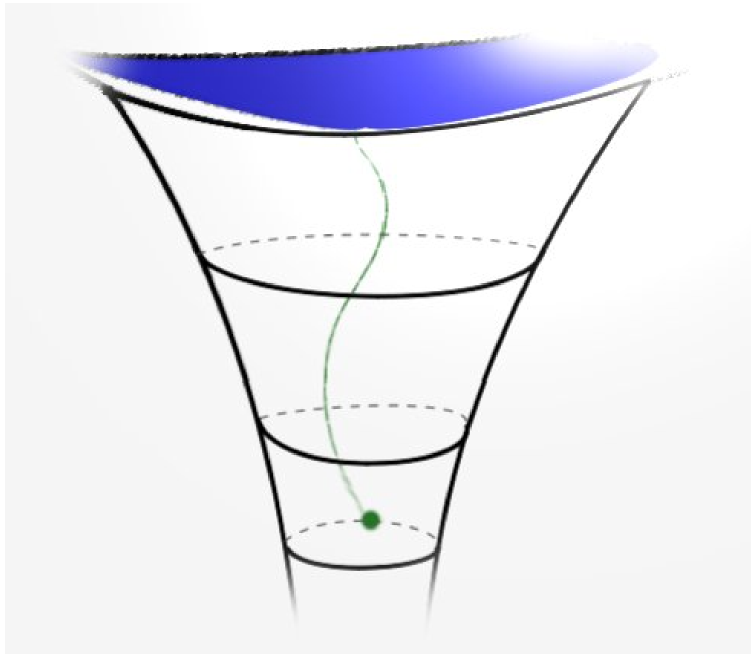


Figure 2.1: A D3-brane which is stabilized far from the tip of the conifold by non-perturbative effects on a bulk cycle receives contributions to the soft masses from a number of sources: uplifted, previously AdS supersymmetric mass-splittings, direct contributions from the uplift pontetial, and by superpotential couplings between the volume modulus and the D3-brane.

2.2.4 Lessons from the toy model

Our explicit computation demonstrates that when including the effects of moduli stabilization, even a 'visible sector' separated from the supersymmetry breaking by a highly warped region can acquire substantial soft masses, primarily through B terms induced by the nonperturbative superpotential.

We now briefly show how the supersymmetry-breaking mass splittings in the visible sector can be interpreted as stemming from an array of different effects. The divisions presented here are somewhat artificial, but can be helpful in tracing the origin of the soft masses; we will give a unified treatment in §2.3. First, supersymmetric Bose-Fermi mass splittings in AdS_4 will be incompatible with Minkowski-space supersymmetry, so that the uplift transforms AdS-supersymmetric mass splittings into non-supersymmetric Minkowski mass splittings.

Second, the uplift induces small shifts of the vevs; in particular, the shift of the (lowest component of the) Kähler modulus superfield results in a nonvanishing vev for the corresponding F-term F_T. Then, nonperturbative superpotential couplings between the visible sector and the Kähler modulus T of the form (2.4) give rise to soft masses in the visible sector of the form

$$\delta\mathcal{L}_{soft} = -a\,O_{vis}\,e^{-aT}F_T + c.c.\,. \tag{2.54}$$

Finally, direct communication through the bulk can induce non-holomorphic masses, which in the four-dimensional theory arise from Kähler potential couplings of the form

$$\delta K = \int d^4\theta\, Q^\dagger Q X^\dagger X\,, \tag{2.55}$$

with X a spurion for hidden-sector supersymmetry breaking. However, as we have reviewed, [46] demonstrated that couplings of this third type are suppressed in *warped* backgrounds, by the gravity dual of conformal sequestering.

In our explicit example, the leading contribution arose from soft masses of the first kind, i.e. AdS-supersymmetric mass splittings transposed to Minkowski space. Soft masses of the second kind, i.e. soft masses arising from F-terms induced by uplifting, turned out to be subleading in $1/(aU)$, $1/B$, cf. the discussion surrounding (2.35), but we expect that in more general models these masses will not be negligible. We now turn to applying these considerations to a more realistic model.

2.3 Soft Terms in Realistic Models

The results of the preceding sections have shown that in a simple string theory toy model for the visible sector — a single D3-brane — soft masses of order $m_{3/2}$ are induced by superpotential cross-couplings. This model differs from the MSSM in two very important respects. First, the 'visible sector' scalar superfields, i.e. the superfields whose lowest components are the D3-brane coordinates z_i, appeared in a rather complicated way in the superpotential (2.25). Of course, in the MSSM, gauge invariance and R-symmetry severely restrict the form of the superpotential, and the only allowed terms up to cubic order in the visible fields are those in (2.4). As we will see, the structure in (2.4) provides the squarks and sleptons with some degree of protection from superpotential de-sequestering, but the Higgs sector enjoys no such protection a priori.[12]

Second, while some visible sector fields in the model in §2.2 obtained significant vacuum expectation values and F-terms, this is not a desired feature of a realistic visible sector (before electroweak symmetry breaking). In the models that we will consider subsequently, the vanishing of the visible sector F-terms will be an assumption, while in a complete string theory construction it should be the outcome of a computation.

In this section we instead assess the effect of superpotential cross-couplings for MSSM-like models that we assume are embedded in a string compactification through an otherwise unspecified D-brane construction. The details of the moduli stabilization scenario have important consequences for the resulting soft terms. In §2.3.1 we study the KKLT model [26], assuming that the

[12] It is conceivable that a mechanism that solves the tree-level μ-problem could also protect the Higgs sector against superpotential de-sequestering, but this is highly model-dependent.

supersymmetry-breaking sector is localized far down a warped throat, so that the Kähler potential is of the form (2.10), while in §2.3.2 we study the Large Volume Scenario (LVS), where extended no-scale has been argued to imply a special form of the metric on the visible sector moduli space, as discussed in §2.1.4. In both cases, soft masses can be computed within a general framework that we now outline.

Along the lines of [102], let us consider a supergravity theory containing visible sector fields C^a and a modulus field T. (We will consider multiple moduli fields later.) In an expansion around zero vacuum expectation values for the visible sector scalars, the superpotential and Kähler potential can be expanded as

$$W = \widehat{W}(X) + \mu(X) H_u H_d + \frac{1}{6} Y_{abc}(X) C^a C^b C^c + \dots \,, \tag{2.56}$$

$$K = \widehat{K}(X, \bar{X}) + \tilde{K}_{a\bar{b}}(X, \bar{X}) C^a \bar{C}^{\bar{b}} + \dots \,, \tag{2.57}$$

where we are assuming that no holomorphic or anti-holomorphic terms such as $Z(X, \bar{X}) H_u H_d + h.c.$ are present[13].

As discussed in §2.2.2, a supersymmetry-breaking Minkowski vacuum is obtained by adding an additional uplift contribution to the scalar potential. This uplift potential is in general dependent on both the visible sector fields and the Kähler moduli.

Then in order to compute the soft scalar masses [77] (not yet canonically normalized, which we indicate with the hats) in (2.11) one has to expand the total potential $V_{tot} = V_F + V_{up}$ (where the standard supergravity F-term potential

[13]The effects of $Z \neq 0$ are discussed e.g. in [63].

V_F is given in (A.9)),

$$
\begin{aligned}
B\hat{\mu} &= \nabla_{h_u}\nabla_{h_d}(V_F + V_{up})\Big|_{C=0}\,, && (2.58)\\
\hat{M}^2_{a\bar{b}} &= \nabla_a\nabla_{\bar{b}}(V_F + V_{up})\Big|_{C=0} - m_{ab}K^{b\bar{c}}m_{\bar{c}\bar{b}}\Big|_{C=0}\,, && (2.59)\\
\hat{A}_{abc} &= \nabla_a\nabla_b\nabla_c(V_F + V_{up})\Big|_{C=0}\,. && (2.60)
\end{aligned}
$$

Here m_{ab} denotes the fermion masses, which in general are given by equation (A.13) and are not corrected by the uplift potential. The contributions to the unnormalized soft terms from V_F were obtained a long time ago [102, 63, 64], and in this notation can be written as

$$
\begin{aligned}
\nabla_{h_u}\nabla_{h_d}V_F &= e^{\widehat{K}/2M_{Pl}^2}\mu\left[F^m\left(\partial_m\log\mu + \frac{\hat{K}_m}{M_{Pl}^2} - \Gamma^{h_u}_{mh_u} - \Gamma^{h_d}_{mh_d}\right) - \frac{\widehat{\overline{W}}}{|\widehat{W}|}m_{3/2}\right]\,, && (2.61)\\
\nabla_a\nabla_{\bar{b}}V_F &- m_{ab}K^{b\bar{c}}m_{\bar{c}\bar{b}} = \left(\frac{F^mK_{m\bar{n}}\bar{F}^{\bar{n}}}{M_{Pl}^2} - 2m_{3/2}^2\right)\tilde{K}_{a\bar{b}}\\
&\qquad -F^{\bar{m}}F^n\left(\partial_n\bar{\partial}_{\bar{m}}\tilde{K}_{a\bar{b}} - \Gamma^{\bar{d}}_{\bar{m}\bar{a}}K_{c\bar{d}}\Gamma^c_{nb}\right)\,, && (2.62)\\
\nabla_a\nabla_b\nabla_cV_F &= e^{\widehat{K}/2M_{Pl}^2}F^m\left[\partial_mY_{abc} + \frac{\widehat{K}_m}{M_{Pl}^2}Y_{abc} - \left(\Gamma^d_{ma}\,Y_{dbc} + (a\leftrightarrow b) + (a\leftrightarrow c)\right)\right]\,, && (2.63)
\end{aligned}
$$

where every expression is to be evaluated at $C = 0$ and we have specialized to Minkowski space. The gravitino mass is given by $m_{3/2} = \frac{|W|}{M_{Pl}^2}e^{K/2M_{Pl}^2}$ and we have also defined[14] $F^m = e^{K/2M_{Pl}^2}K^{m\bar{n}}\bar{D}_{\bar{n}}\overline{W}$ for m, n taking values over all relevant moduli fields. Notice that corrections to the μ and Y terms in the superpotential (2.16) lead to corrections in the holomorphic quadratic and cubic terms (2.61) and (3.84), but not in (2.62).

Turning our attention to the uplift potential, we will assume, as we did in §2.2.2, a term of the form

$$
V_{\rm up} = De^{\frac{2K}{3M_{Pl}^2}} = \frac{D}{U^2}\,, \qquad (2.64)
$$

[14]With this definition of F^m, care is needed in lowering and raising indices, since $F_m \equiv D_mW = e^{-K/2M_{Pl}^2}K_{\bar{n}m}\bar{F}^{\bar{n}}$.

where D is a constant, corresponding for example to supersymmetry breaking by an anti-D3-brane. The vanishing of the visible sector vevs together with the absence of visible sector gauge-invariant linear terms in the Kähler potential implies that $K_a = 0$ (note that this is quite different from the toy model we considered earlier), so the only contribution to the soft terms from the uplift potential is to the soft scalar masses:

$$\nabla_a \nabla_b \nabla_c V_{up} = \nabla_a \nabla_b V_{up} = 0\,, \tag{2.65}$$

$$\nabla_a \nabla_{\bar{b}} V_{up} = \frac{2\tilde{K}_{a\bar{b}}}{3M_{Pl}^2} V_{up}\,. \tag{2.66}$$

In order to proceed further we need to specify a scenario for Kähler moduli stabilization.

2.3.1 KKLT stabilization

Soft terms in the KKLT scenario have been previously studied by a number of authors [78, 49, 73]. In this section we investigate the effects of superpotential de-sequestering for a MSSM-like visible sector. To be concrete, in a theory with one volume modulus T and visible sector chiral superfields $C^a \equiv Q^i, u^i, d^i, L^i, e^i$, H_u, H_d, the separability of the f function (which can be justified e.g. by warped sequestering [46]), implies that the lowest-dimension terms in the Kähler potential are (again omitting purely holomorphic terms, as in (2.57))

$$K = -3M_{Pl}^2 \ln\left(T + \bar{T} - \frac{1}{3M_{Pl}^2}\sum_a C^a \bar{C}^a\right) = -3M_{Pl}^2 \ln U. \tag{2.67}$$

The Peccei-Quinn symmetry of the axion field which is the imaginary part of T is only broken by nonperturbative effects, so the leading superpotential terms consistent with Standard Model gauge symmetry and with the PQ symmetry are

$$\begin{aligned} W &= W_0 + W^{\text{tree}} + \\ &+ \mathcal{A}_0\Big(1 + \frac{\hat{\mu}}{M_{Pl}^2}H_uH_d + \frac{\lambda^u_{ij}}{M_{Pl}^3}Q^iu^jH_u + \frac{\lambda^d_{ij}}{M_{Pl}^3}Q^id^jH_d + \frac{\lambda^l_{ij}}{M_{Pl}^3}L^ie^jH_d\Big)e^{-aT}\,. \end{aligned} \tag{2.68}$$

The detailed structure of the tree-level superpotential,

$$W^{\text{tree}}(C) = \lambda^{u,\text{tree}}_{ij}Q^iu^jH_u + \lambda^{d,\text{tree}}_{ij}Q^id^jH_d + \lambda^{l,\text{tree}}_{ij}L^ie^jH_d + W^{\text{tree}}(H_u, H_d)\,, \tag{2.69}$$

is dependent on the exact realization of the visible sector, but it is generally true that W^{tree} does not depend on the modulus T. We also remark that $\mathcal{A}_0$ may in general depend on additional hidden sector fields, but for our purposes it suffices to treat $\mathcal{A}_0$ as a constant. For a discussion of the sizes of the couplings $\hat{\mu}$ and $\lambda^{u,d,l}_{ij}$, see §**??**. Let us consider in turn the various soft terms.

$B\mu$ term

The leading contribution to the $B\mu$ term comes from the F-term potential because of (2.65). Since K is of the sequestered form (2.67), only the first and the last terms in (2.61) contribute, to leading order in $1/U$. The first term is of the form expected from global supersymmetry as the T modulus obtains a non-vanishing F-term vev, while the last term, which is proportional to the gravitino mass, can be interpreted in KKLT as an uplifted, previously AdS-supersymmetric B term. In a perturbative expansion around the supersymmetric solution, we find $F_T \approx \delta X^T \partial^2_{TT}W|_{Z_0} = -\frac{3aW}{U}\delta X^T$, where δX^T is the shift in the real part of the volume modulus due to the uplift. Since $K^{T\bar{T}} = \frac{U^2}{3M_{Pl}^2}$, we find that

$$F^T = e^{K/2M_{Pl}^2}K^{T\bar{T}}\bar{F}_{\bar{T}} = -\frac{3a}{U}\frac{U^2}{3M_{Pl}^2}\delta X^T e^{K/2M_{Pl}^2}\overline{W} = -aU\frac{\overline{W}}{|W|}m_{3/2}\,\delta X^T. \tag{2.70}$$

The leading contribution to the shift in the vacuum expectation value of T is given by $\delta T = -\partial_N V_{up}(\partial^2_{TN}V_{tot})^{-1}$, which in the case of vanishing visible sector vevs evaluates to $\delta T = \frac{2}{a^2 U}$ for the dimensionless modulus. It follows that the dimension one F-term F^T is

$$F^T = -\frac{2}{a^2 U} a U \frac{\overline{W}}{|W|} m_{3/2} = -\frac{2}{a} \frac{\overline{W}}{|W|} m_{3/2}. \tag{2.71}$$

Neglecting a possible contribution to the μ term from the tree-level superpotential[15] W^{tree}, we have $\partial_T \log \mu(T) = -a$, and the un-normalized $B\mu$ term, $B\hat{\mu}$, becomes

$$\begin{aligned} B\hat{\mu} &\simeq e^{\hat{K}/2M_{Pl}^2} \mu \left(F^m \partial_m \log \mu - m_{3/2} \frac{\overline{W}}{|W|} \right) = e^{\hat{K}/2M_{Pl}^2} \mu \, m_{3/2} \frac{\overline{W}}{|W|} \\ &= -m_{3/2}^2 \frac{3\hat{\mu}}{aU}. \end{aligned} \tag{2.72}$$

After canonically normalizing the Higgs field kinetic terms, the physical $B\mu$ term becomes

$$B\mu \simeq -\frac{3\hat{\mu}}{a} m_{3/2}^2. \tag{2.73}$$

We emphasize that this $\hat{\mu}$ is the coefficient in the nonperturbative contribution 2.68.

Soft non-holomorphic masses

The soft masses receive contributions both from the F-term potential (2.62) and from the uplift potential (2.66). If the uplift had been achieved purely through the F-term vevs of the moduli fields, so that $F^m \bar{F}^{\bar{n}} K_{m\bar{n}} = 3m_{3/2}^2 M_{Pl}^2$, the first line of the right-hand side of (2.62) could have been written simply as $+m_{3/2}^2 \tilde{K}_{a\bar{b}}$ in Minkowski space. In KKLT, however, the uplift is obtained by adding an explicit uplift potential, so for the F-terms in equation (2.71) we instead find

$$F^T K_{T\bar{T}} \bar{F}^{\bar{T}} = \frac{4}{(aU)^2} \cdot 3m_{3/2}^2 M_{Pl}^2. \tag{2.74}$$

[15] A tree-level μ term of $O(v)$ would result in corrections to this estimate of the order of $\frac{v}{U^{1/2} m_{3/2}}$.

These terms are then a priori subleading with respect to the contribution proportional to $m_{3/2}^2$. However, after adding the contribution from the uplift potential as in (2.66),

$$\nabla_a \nabla_{\bar{b}} V_{up} = \frac{2V_{up}}{3M_{Pl}^2} \tilde{K}_{a\bar{b}} = 2m_{3/2}^2 \tilde{K}_{a\bar{b}} \,, \tag{2.75}$$

the leading contribution to the sfermion and Higgs un-normalized soft masses cancels precisely. Retaining the first subleading term, one finds

$$\hat{M}_{a\bar{b}}^2 \quad \simeq \quad \left[2 - 2 + \frac{8}{(aU)^2}\right] m_{3/2}^2 \tilde{K}_{a\bar{b}} \,. \tag{2.76}$$

The leading-order contribution shows an exact cancellation between two terms of very different physical origin. While the negative contribution to the soft masses can be interpreted as mass splittings that were supersymmetric in AdS, the positive contribution is a direct effect of the supersymmetry-breaking uplift potential. The exact cancellation is a consequence of the fact that the uplift potential for an anti-D3 brane is given by $\frac{D}{U^n}$ for $n = 2$.[16] For $n \neq 2$ — as can be expected for uplifts corresponding to objects extended in the internal dimensions — the soft masses would be of the order of $m_{3/2}$. In conclusion, for the Kähler potential in (2.67), the superpotential in (2.68) and for anti-D3-brane uplifting, we find that the total soft mass for the normalized visible sector fields can be written as

$$M_{a\bar{b}}^2 = \frac{8}{(aU)^2} m_{3/2}^2 \, \delta_{a\bar{b}} \,, \tag{2.77}$$

which is flavor-universal and does not induce additional flavor-changing neutral currents.

[16] The uplifting was described in terms of a spurion superfield in [78], where the cancellation for $n = 2$ was also noticed. An off-shell formulation of this sort is not required for our purposes.

A terms

Finally, let us consider the soft trilinear A terms. We first note that $K_T = -3M_{Pl}^2/U$, $\Gamma_{Ta}^c = -\delta_a^c \frac{1}{U}$, and the Yukawa coupling can be written as $Y_{uij} = \mathcal{A}_0/M_{Pl}^3 e^{-aT} \lambda_{ij}^u + \lambda_{ij}^{u,\mathrm{tree}}$ for the up-type quark superfields and similarly for the down-type quarks and for the leptons. Using this in the expression (3.84) for the un-normalized A terms results in an exact cancellation of everything but the first term, as expected from the extended no-scale discussion in §2.1.4. Thus, the un-normalized scalar trilinear A term is given by

$$\begin{aligned} \hat{A}_{uij} &= e^{\hat{K}/2M_{Pl}^2} F^m \partial_m Y_{uij} \\ &= -\frac{6}{aU} \frac{m_{3/2}}{M_{Pl}} m_{3/2}\, \lambda_{ij}^u\,. \end{aligned} \tag{2.78}$$

After electroweak symmetry breaking $\langle H_{u,d}^0 \rangle \equiv v_{u,d}$, the A terms will contribute to the sfermion masses. Unless A is real and has very special flavor structure, this will induce FCNC and CP violation. A useful quantity for which model-independent experimental constraints are available [79] is $\delta \equiv \Delta_{ab}/M_{\mathrm{soft}}^2$, where Δ_{ab} is the flavor-off-diagonal contribution to the propagator of the sfermions in the basis in which the couplings to neutral gauginos are flavor diagonal, and M_{soft} is the average sfermion mass. In principle the soft terms we compute are valid at some high supersymmetry breaking scale and should be run down to the scales where experiments are performed using RG equations. The largest running appears in the third-generation quarks because of the large top Yukawa coupling. Since the experimental data we focus on does not involve the third generation, we ignore the effects of RG running.

E.g. for canonically normalized up-type squarks we find

$$\delta \equiv \frac{\Delta_{ab}}{M_{\mathrm{soft}}^2} = -\frac{6U^{1/2}}{a} \left(\frac{m_{3/2}}{M_{Pl}}\right)\left(\frac{m_{3/2} v_u}{M_{\mathrm{soft}}^2}\right) \lambda_{ij}^u\,. \tag{2.79}$$

For a numerical estimate, as in §2.2.3, we take $m_{3/2} \sim aUM_{\text{soft}}$ [78, 49], $aU \sim 66$ and $a = \frac{2\pi}{32}$. We find

$$\delta^{u,d} \sim 10^{-11} \left(\frac{v_{u,d}}{100\text{GeV}} \right) \lambda^{u,d}_{ij} \,, \tag{2.80}$$

which for $|\lambda^{u,d}_{ij}| \sim 1$ is at least four orders of magnitude away from any experimental constraint from FCNC and CP violation [79].

Summary of KKLT phenomenology

In sum, in the single volume modulus KKLT scenario, flavor diagonal squark masses of the order of $\frac{1}{(aU)} m_{3/2}$ are induced, resulting in negligible flavor violation [78, 49]. For multiple Kähler moduli, this conclusion holds as long as the Kähler potential is of the sequestered form (2.10). Superpotential de-sequestering in the Higgs sector can lead to a significant $B\mu$ term of order $m^2_{3/2}$, and to supersymmetric masses of order $m_{3/2}$, which can create serious problems for electroweak symmetry breaking.

We note that as the vacuum expectation value of F_T becomes important for the soft terms, sequestering breaks down severely, in that modifications to the compactification located far from the visible sector in the internal dimensions can have a significant impact on F_T, which in turn affects the physics of the visible sector. For instance, the value of F_T in §2.2.3 where the effects of a nearby D3-brane are taken into account is exactly half of that for the KKLT scenario in the absence of the D3-brane, cf. the discussion around equation (2.53). Regarding the D3-brane as a part of the hidden sector, we note that this reduction in F_T leads to a cancellation of the leading-order $B\mu$ term (2.72). Instead of being of order $m^2_{3/2}$, the $B\mu$ term will in this case enter at $O(m^2_{3/2}/U)$. A similar sensitivity to the global compactification — though perhaps not as striking — is a general feature of the contributions proportional to F_T that are induced by superpotential de-sequestering.

2.3.2 The Large Volume Scenario

So far we have only discussed the effects of superpotential de-sequestering in KKLT compactifications, but we expect significant effects whenever a nonperturbative superpotential plays an important role in the stabilization of the moduli. We illustrate this point by turning to the Large Volume Scenario (LVS), one of the most promising areas for successful phenomenology from string compactifications [27]. In LVS, α'-corrections to the Kähler potential are included in addition to the the same nonperturbative superpotential as in KKLT (2.68), both effects breaking no-scale. For Calabi-Yau manifolds of the Swiss-cheese type,[17]

$$\mathcal{V} \simeq \tau_b^{3/2} - \tau_s^{3/2}\,, \tag{2.81}$$

a non-supersymmetric AdS vacuum exists where the overall volume $\mathcal{V}$ is exponentially large, $\mathcal{V} \simeq \tau_b^{3/2} \simeq e^{a\tau_s}$, with $\tau \equiv (T+\bar{T})/2$. Hence in W only the nonperturbative terms for the small volumes τ_s are relevant. So we have

$$\begin{aligned} W &= W_0 + W^{\text{tree}} + \\ &+ \mathcal{A}_0\Big(1 + \frac{\hat{\mu}}{M_{Pl}^2}H_u H_d + \frac{\lambda_{ij}^u}{M_{Pl}^3}Q^i u^j H_u + \frac{\lambda_{ij}^d}{M_{Pl}^3}Q^i d^j H_d + \frac{\lambda_{ij}^l}{M_{Pl}^3}L^i e^j H_d\Big)e^{-aT_s}\,. \end{aligned} \tag{2.82}$$

It is important to notice that LVS crucially requires at least two Kähler moduli, in which case, as opposed to the case of a single overall volume, the separability

[17] To avoid cluttering our notation with another index we write formulae for a single T_s. The generalization to many T_s's is straightforward. Moreover, adding a vanishing four-cycle T_a as in [61, 80] would not change our discussion.

(2.7) of f into hidden and visible sector contributions is far from established. Despite this fact, the milder condition of extended no-scale discussed in §2.1.4 has been argued to hold at least to leading order in four-cycle volumes. Let us review the argument given in [65]. Recall that for a diagonal visible sector matter metric, $\tilde{K}_{a\bar{b}} = \tilde{K}_a \delta_{a\bar{b}}$ (no sum), the physical Yukawa couplings in supergravity are given by

$$Y^{\rm phys}_{abc} = e^{K/2M_{Pl}^2} \frac{Y^{\rm hol.}_{abc}}{\sqrt{K_a K_b K_c}} \, , \tag{2.83}$$

and can be computed from first principles in a given localized model by computing the overlap of the corresponding normalized wavefunctions. In [65], it was argued that locality implies that the physical Yukawa couplings should be independent of the volume modulus to leading order in perturbation theory. Since the *holomorphic* Yukawa couplings are independent of the volume modulus to all orders in perturbation theory by holomorphy combined with the Peccei-Quinn symmetry, this requirement enforces a special form of the metric on the visible sector moduli space: $\tilde{K}_a \sim e^{\widehat{K}/3M_{Pl}^2}$. This leads to the structure studied in §2.1.4 [61],

$$\tilde{K}_{a\bar{b}} = e^{\widehat{K}/3M_{Pl}^2} \kappa_{a\bar{b}} \, , \tag{2.84}$$

with $F^m \partial_m \kappa_{a\bar{b}} = 0$, i.e. $\kappa_{a\bar{b}}$ does not depend on the moduli that get non-vanishing F-term vevs. In [72], the virtue of this condition was emphasized in the context of LVS. Volume-suppressed violations of this form generically lead to non-universal soft masses, as discussed in [81].

Superpotential cross-couplings induce corrections to the $B\mu$ terms and A terms but not to the nonholomorphic sfermion masses M^2. Given (2.65), the corrections $\delta B\mu$ and δA can be computed from the first term on the right hand side of (2.61) and (3.84), respectively. We now consider these contributions in turn.

$B\mu$ term

We want to compute the contribution to the $B\mu$ term from superpotential cross-couplings, which is given by the first term in (2.61). To canonically normalize the visible sector fields we need $\tilde{K}_{a\bar{b}}$. For our purposes, it is sufficient to know it at leading order in the large volume expansion, since there are no precise cancellations that make the subleading orders important. We can hence use the extended no-scale relation [65] and rotate and rescale the fields to have $\tilde{K}_{a\bar{b}} = e^{\widehat{K}/3M_{Pl}^2}\delta_{a\bar{b}}$. Then we find the correction from superpotential cross-couplings to the canonically normalized $B\mu$ term to be

$$\begin{aligned} \delta B\mu &= e^{\widehat{K}/6M_{Pl}^2} F^m \partial_m \mu && (2.85) \\ &= -\frac{a}{\mathcal{V}^{1/3}} F^{T_s} e^{-a\tau_s} \frac{\mathcal{A}_0 \hat{\mu}}{M_{Pl}^2} \, . && (2.86) \end{aligned}$$

For an estimate, we assume $|\mathcal{A}_0| \sim M_{Pl}^3$, $|\hat{\mu}| \sim 1$ (we will discuss this assumption in §**??**), and use the facts that in LVS, the volume at the minimum of the potential is $\mathcal{V} \simeq e^{a\tau_s}$ and $|F^{T_s}| \simeq m_{3/2}$. Keeping track only of volume factors we find

$$\delta B\mu \sim \frac{M_{Pl}^2}{\mathcal{V}^{7/3}} \, . \qquad (2.87)$$

For successful electroweak symmetry breaking, $B\mu$ should not be far from the weak scale. Hence, unless this contribution is absent, (3.22) puts a strong lower bound on the size of the overall volume: $\mathcal{V} \gtrsim 10^{14}$ in string units.

A terms

Now let us turn to the contribution to the soft A terms from superpotential cross-couplings. This is given by the first term in (3.84). After canonically normalizing the visible sector fields using $\tilde{K}_{a\bar{b}} = e^{\widehat{K}/3M_{Pl}^2}\delta_{a\bar{b}}$ [65] at leading order (since again there is no relevant cancellation for the terms we are computing), we have

$$\begin{aligned} \delta A_{uij} &= e^{-\widehat{K}/2M_{Pl}^2} e^{\widehat{K}/2M_{Pl}^2} F^m \partial_m Y_{uij} \\ &= -\frac{\mathcal{A}_0 \lambda^u_{ij}}{M_{Pl}^3} a F^{T_s} e^{-a\tau_s}\,, \end{aligned} \tag{2.88}$$

and similarly for δA_{dij} and δA_{lij}. As we saw in §2.3.1, an efficient way to estimate the phenomenological effect of this correction,[18] e.g. in terms of FCNC and CP violation, is to compute the parameter δ defined in (2.79). Using $e^{a\tau_s} \simeq \mathcal{V}$ as above, we write

$$\delta = \frac{\mathcal{A}_0 \lambda_{ij}}{M_{Pl}^3} \frac{a}{\mathcal{V}} \frac{v F^{T_s}}{M_{\rm soft}^2}\,. \tag{2.89}$$

Let us focus just on the scaling with the overall volume, neglecting factors of a and T_s and setting $|F^{T_s}| \simeq m_{3/2}$. The size of $M_{\rm soft}$, being the average value of the relevant soft terms, is still a matter of debate [82, 83, 61], so we parameterize it as

$$M_{\rm soft}^2 \sim \frac{m_{3/2}^2}{\mathcal{V}^n}\,, \tag{2.90}$$

where n has been claimed to be $\{0, 1/3, 1 \text{ or } 2\}$ in [82, 83, 61] respectively. Putting things together we find

$$\delta \sim \mathcal{V}^n 10^{-16} \left(\frac{v}{100\,\mathrm{GeV}}\right)\left(\frac{\mathcal{A}_0}{M_{Pl}^3}\right)\lambda_{ij}\,. \tag{2.91}$$

One of the strongest experimental constraints on δ comes from bounds on $\mu \to e\gamma$ and gives $|(\delta^l_{12})_{LR}| < 2 \cdot 10^{-6}$ [79]. Other strong constraints arise due to the CP

[18] Again, for the quantities of interest here we expect the RG evolution to give only negligible corrections.

violation induced by $\mathrm{Im}\,\delta$, e.g. from electric dipole moments. Figures of merit are $|\mathrm{Im}(\delta^d_{11})_{LR}| < 3 \cdot 10^{-6}$, $|\mathrm{Im}(\delta^u_{11})_{LR}| < 6 \cdot 10^{-6}$ and $|\mathrm{Im}(\delta^l_{11})_{LR}| < 4 \cdot 10^{-7}$.

Unless the nonperturbative correction is real and respects flavor (more on this point in §**??**), we find the following upper bound on the size of the overall volume in LVS:

$$\mathcal{V} < 10^{10/n} \left(\frac{100\,\mathrm{GeV}}{v} \right)^{1/n} . \tag{2.92}$$

For a crude estimate, here we have assumed $|\lambda_{ij}| \sim 1$ and $\mathcal{A}_0 \sim M^3_{Pl}$. For $n = 0, 1/3$, as claimed in [82, 83], respectively, this bound is irrelevant. On the other hand, in the case considered in [61] for $|\lambda_{ij}| \approx 1$ and $\mathcal{A}_0 \approx M^3_{Pl}$ one finds the upper bounds $\mathcal{V} < 10^{10}$ and $\mathcal{V} < 10^5$ for $n = 1$ and $n = 2$, respectively. Since in [61] it was argued that the smallest possible volume is $\mathcal{V} \sim 10^{6-7}$ in string units, for $n = 2$ there might already be some tension with experiments due to superpotential de-sequestering. A careful analysis keeping track of all $O(10)$ factors neglected in the above estimate would be desirable. Finally, notice that this bound is in contradiction with the one obtained from the size of the $B\mu$ term (3.22), leading to an inconsistency unless one of the two contributions is forbidden by some additional mechanism. Summarizing, the bound (2.92) shows that data on CP violation and FCNC could be used to rule out an interesting region of parameter space.

2.3.3 On nonperturbative corrections to visible sector superpotentials.

Our discussion so far has assumed that because nonperturbative superpotential couplings between the visible sector and the Kähler moduli are not forbidden by known symmetries, these couplings are in fact present, and are not simply proportional to the tree-level Yukawa couplings. That is, if the visible-sector superpotential includes terms of the form[19]

$$W_{vis} = \lambda_{ij}^{u,\text{tree}} Q^i u^j H_u + \lambda_{ij}^{u} Q^i u^j H_u \, e^{-aT} \,, \tag{2.93}$$

then our working assumption has been that $\lambda_{ij}^{u,\text{tree}}$ and λ_{ij}^{u} are not proportional.

This assumption is strongly supported by the abundance of examples in the literature in which string instantons or D-brane instantons give rise to Yukawa couplings that are forbidden in perturbation theory (see [84] for a comprehensive review in the context of intersecting D6-branes in type IIA orientifolds, and [85] for a discussion of D-brane instantons stretched between the visible and hidden sectors.) In F-theory GUT models, contributions from nonperturbative effects on D7-branes have been argued to solve the 'rank problem' of the tree-level Yukawa couplings [86], which obviously requires an adjustment of the flavor structure. Thus, flavor violation by nonperturbative effects is well-attested in string-theoretic realizations of the MSSM.

It would be interesting to obtain further details on the form of nonperturbative superpotential couplings by direct computation in string models. A full treatment of this point is beyond the scope of the present work, but we now outline what needs to be done to evaluate this.

A terms

To acquire a more detailed picture of nonperturbative contributions to *A* terms in type IIB compactifications, we now examine an analogous computation for

[19] To simplify our expressions, in this section we will focus on u-type quarks; the extension to the remaining fermions is trivial.

adjoint open string fields [67]. There, the moduli-dependent string S-matrix of D7-brane gauge field vertex operators $\mathcal{V}_{A_\mu}$ was considered:[20]

$$\langle \mathcal{V}_{A_\mu} \mathcal{V}_{A_\nu} \rangle_{\text{background } \phi} \; = \; \text{function of moduli } S, T, U, \phi \; . \tag{2.94}$$

From this string correction to the physical gauge couplings one extracts a correction to the holomorphic gauge kinetic function of gauge fields on D7-branes:

$$f_{1-\text{loop}} = -2 \ln \vartheta_1(\phi, U) + \ldots \tag{2.95}$$

where the omitted terms are independent of ϕ and $\vartheta_1(\phi, U)$ is a Jacobi theta function. Now, substituting $f_{1-\text{loop}}$ in the nonperturbative superpotential on D7-branes,

$$W = \mathcal{A} e^{-a(f_{\text{tree}} + f_{1-\text{loop}})} \, , \tag{2.96}$$

we obtain

$$W = \mathcal{A}(\phi) e^{-aT} \, , \tag{2.97}$$

where

$$\mathcal{A}(\phi) = (\vartheta_1(\phi, U))^{2a} \; . \tag{2.98}$$

This is a toy model, but it was shown in [68] that analogous expressions appear in the backgrounds of interest. The key point here is simply that $\mathcal{A}$ inevitably depends on ϕ, and the dependence is not in any way negligible.

We expect a similar calculation to be feasible also for chiral matter fields, though more challenging. In that case, one would expand the function $\mathcal{A}$ in gauge-invariant operators,[21]

$$\mathcal{A}(\phi^i) = \mathcal{A}_0 + \mathcal{A}^u_{ij} Q^i u^j H_u + \ldots \, , \tag{2.99}$$

[20] To be precise, the string S-matrix is only obtained as an explicit function when the world-sheet moduli are integrated over, as in [67]. See also recent related work in [87].

[21] A variation on this is quite common in D-brane models: T may be charged under an anomalous $U(1)$, so that $A(\phi)$ must also transform under the $U(1)$. To exclude this possibility, we will assume any anomalous $U(1)$'s are broken near the string scale, as is often the case.

where it is understood that these are small fluctuations around the final, non-supersymmetric, minimum.[22] We cannot turn on a background for chiral fields, so this coupling needs to be probed by the S-matrix due to the following five-point function:

$$\langle \mathrm{tr}(\mathcal{V}_{A_\mu}\mathcal{V}_{A_\nu})\mathrm{tr}(\mathcal{V}_{\phi_i}\mathcal{V}_{\phi_j}\mathcal{V}_{\phi_k})\rangle\,, \tag{2.100}$$

where the traces are over the respective gauge groups. This is a double trace operator and so will appear at loop level. The vertex operators $\mathcal{V}_{\phi_i}$ for chiral fields are given by the usual vertex operators for open string scalars ϕ but with boundary changing operators σ that change boundary condition from one brane stack i to the next stack j. The cylinder diagram one needs to compute is shown in the left panel of figure 2.2. As shown in the right panel, this can factorize onto some closed string field, call it X.[23] If the field X appears in the moduli-dependent superpotential as $\mathcal{A}^u_{ij}(X)Q^i u^j H_u$ (this is sometimes described as X "carrying flavor"), the resulting coupling will be problematic in general, as we do not expect $\lambda^{u,\mathrm{tree}}_{ij} \propto \lambda^u_{ij}$.

However, let us mention a possible mechanism by which the nonperturbative contributions might respect the flavor symmetry preserved by the tree-level couplings. In intersecting brane models, and for some models with branes at singularities (e.g. $\mathcal{O}(-3)_{\mathbb{P}^2} \sim \mathbb{C}^3/\mathbb{Z}^3$) the Yukawa couplings arise as triple intersections between three brane stacks. Consider the D3-brane part of the right panel of figure 2.2; without the closed string insertion X, the tree-level three-point diagram is what generates Yukawa couplings y_{ij} in the first place. It is possible that in the low-energy limit, the coupling λ^u_{ij} that is generated could

[22]Another example of charged fields analogous to (2.99) is [88].
[23]Because of the PQ symmetry, X cannot be T.

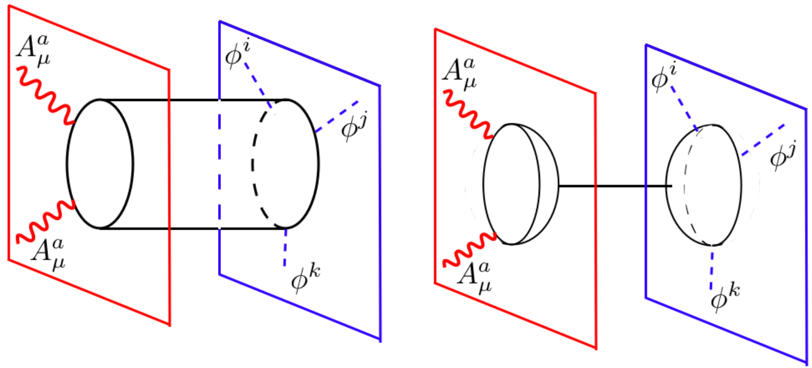


Figure 2.2: D7-branes on the left of cylinder (in red), D3-branes on the right of cylinder (in blue). Left panel: Five-point function analogous to the two-point function considered in [67]. Right panel: the factorized limit. The flavor structure of λ^u_{ij} may in principle be determined by that of the tree-level (disk) three-point function.

satisfy

$$\lambda^u_{ij} = c\,\lambda^{u,\text{tree}}_{ij} \tag{2.101}$$

for some constant c, so that no new flavor-changing effects are induced. However, a much more detailed investigation would be required to establish a mechanism along these lines.

In some circumstances, a mechanism ensuring the smallness of the tree-level Yukawas can also control the structure of λ^u_{ij}, but it remains an open question to embed such a mechanism in a string model. We conclude that there is no evidence that the couplings in (2.99) should vanish, nor is there currently a compelling argument that these couplings should generically preserve the tree-level flavor structure.

μ terms

It is not inconceivable that whatever physics causes the vanishing of the tree-level μ-term also requires a large suppression of the constant $\hat{\mu}$ of (2.68). The μ term is often prohibited at a high scale by a continuous PQ symmetry that leaves a remnant discrete symmetry. In such a case, one should ask whether T is charged under the PQ symmetry: if not, then the PQ symmetry forbids non-perturbative corrections to the (prohibited) tree-level μ term. If instead T carries a PQ charge, one would have to explain why the moduli-stabilizing superpotential itself is not forbidden (see also [89]). This question depends in some detail on the particular realization of the MSSM, and as such is beyond the scope of this work.

2.4 Conclusions

Higher-dimensional locality provides a promising organizing principle for the suppression of the phenomenologically dangerous flavor-violating soft terms of gravity-mediated supersymmetry breaking. Although locality in barren extra dimensions leads to sequestering, moduli-mediated couplings in the Kähler potential prevent sequestering in generic unwarped compactifications of string theory [44], even after moduli stabilization [45]. Strong warping suppresses Kähler potential couplings [46] via the gravity dual of conformal sequestering, but we have shown that nonperturbative stabilization introduces new superpotential couplings between the Kähler moduli and the visible sector. As analogous couplings are known to violate tree-level flavor symmetries in many examples, it is reasonable to expect flavor violation in this case as well.

In a very simple and explicit toy model involving a D3-brane in a conifold region of a KKLT compactification, we showed that some of the soft scalar masses induced by the superpotential cross-couplings are of order the gravitino mass. Therefore, supersymmetry breaking in this model does not sequester, in contrast to the positive result obtained by [46] for the corresponding configuration before Kähler moduli stabilization.

Our considerations also apply to more realistic visible sectors. In a KKLT compactification with an MSSM-like sector assumed to be supported on a collection of D-branes separated from the supersymmetry-breaking sector by warping, there are flavor-universal contributions to the sfermion masses of the order of $m_{3/2}/(aU)$. In the Higgs sector, nonperturbative superpotential cross-couplings induce μ and $B\mu$ terms of order $m_{3/2}$ and $m_{3/2}^2$, respectively, with a detailed sensitivity to the global compactification. Thus, in KKLT compactifications, the sequestering described in [46] does not survive moduli stabilization. However, the corrections due to superpotential de-sequestering are rather mild: depending on the full mediation scenario, the corrections to the squark and slepton masses due to superpotential de-sequestering can be made sub-dominant, while for the Higgs sector the nonperturbative superpotential contributions to the μ and $B\mu$ terms are not necessarily fatal but must be properly incorporated, as in [49].

In the case of the Large Volume Scenario — in which the supersymmetry breaking sector is in no way geometrically separated from the visible sector — ten-dimensional locality can still result in a significant suppression of the soft masses with respect to the gravitino mass [61]. However, the nonperturbative superpotential is an essential ingredient in the moduli stabilization, and we found that in certain parameter regimes, soft trilinear A terms induced by nonperturbative superpotential couplings can be dangerously large, so that precision results from flavor physics constrain the model parameters. Intuitively, scenarios with large hierarchies between $m_{3/2}$ and M_{soft} are the most vulnerable to small corrections from nonperturbative superpotential cross-couplings, and indeed we found a flavor and CP problem only in the scenario of [61], where such a hierarchy is present, while non-negligible corrections to the $B\mu$ term are to be expected in essentially all scenarios.

Our work sharpens the criteria for sequestered supersymmetry breaking in a string compactification: examination of the Kähler potential alone is insufficient when nonperturbative superpotential terms control the stabilization of moduli. Two additional tasks are required: one should provide a mechanism that controls nonperturbative contributions to $B\mu$, and one must ensure that any flavor-violating nonperturbative contributions to the A terms are consistent with experiment.

A very interesting task for the future is the construction of an explicit, realistic visible sector in string theory for which the soft masses are sequestered.

CHAPTER 3

TOWARDS CONSTRAINING AFFLECK-DINE BARYOGENESIS

3.1 Introduction

While several ostensibly viable mechanisms for generating the observed baryon asymmetry of the universe have been proposed, which of them — if any — is actually responsible for the discrepancy between the densities of matter and antimatter is not known. A beautiful mechanism of baryogenesis which appears very natural in models with supersymmetry was proposed long ago by Affleck and Dine. In the seminal paper [93], it was noticed that baryogenesis may proceed through the dynamics of flat directions, which in a supersymmetric theory, like the MSSM, generically exist in abundance before supersymmetry is broken. These flat directions may lead to several important cosmological consequences (for reviews see [37, 94]), out of which baryogenesis is perhaps the most spectacular.

The Affleck-Dine scenario provides a robust mechanism for baryogenesis and can easily produce a significant baryon number, large enough to reproduce the observed ratio of baryons over photons,

$$n_B/n_\gamma \simeq 6 \cdot 10^{-10}\,, \tag{3.1}$$

even in the presence of late-time entropy releases. Intriguingly, the viability of the mechanism is contingent upon the structure of Planck-suppressed operators in both the Kähler potential and the superpotential, thus providing a window of sensitivity to high-scale physics like string theory. For several interesting examples of inflation in string theory where these operators can be computed, Affleck-Dine baryogenesis is impossible. Therefore, if this mechanism would be observationally confirmed, it would provide important information guiding string theory constructions of inflation and the standard model.

A number of correlated predictions of the Affleck-Dine mechanism have been noted, including the prediction of so called Q-balls [95] and — as have received much attention recently — the ease with which the near equivalence of the abundance of dark matter and baryons can be explained in this framework [96], see also [37]. Nonetheless, it can been argued that due to its apparent robustness together with the wide range of possible resulting baryon numbers, the scenario can be quite hard to falsify, thus making it less attractive as a physical theory.

The purpose of §3 is to further explore the correlated predictions on the Affleck-Dine scenario. In doing so, we give a clear geometric characterization of the conditions under which the mechanism is viable in §3.2, and exploit the nontrivial structure of $\mathcal{N} = 1$ supergravity to extract correlated predictions for various couplings in the Lagrangian in §3.4.

Furthermore, a potentially interesting consequence of the Affleck-Dine scenario is the 'backreaction' of the flat direction on the inflaton potential, and in §3.3 we discuss in detail how this can give rise to constraints on the parameters of the model. These constraints necessarily involve multi-field inflation, but here we use a simplified single-field model to give a rough estimate its relevance. The strongest constraints are obtained from the anomalies in the temperature anisotropies of the cosmic microwave background at $l \sim 20$ and $l \sim 40$, which from the temperature measurements alone are only marginally significant. Future observations of the polarization of the cosmic microwave background will decisively determine the severity of the constraints that can be imposed on the Affleck-Dine mechanism from precision cosmology.

We discuss this consequence for both thermal and non-thermal initial conditions for the flat directions, and in both cases the effect is observationally relevant only if a flat direction is displaced from the global minimum of the potential at the time when cosmological scales left the horizon. This can be avoided in a number of ways, e.g. by inflation persisting for much longer than the around sixty e-folds required to solve the horizon problem.

3.1.1 The Affleck-Dine mechanism

For Affleck-Dine baryogenesis to be successful, it is crucial that the scalar components ψ^a of one or several chiral superfields (denoted Ψ^a) parametrizing gauge invariant, renormalizably flat directions, obtain large vacuum expectation values (vevs) with nontrivial phases during inflation. However, there are a number of effects that can trap the flat direction at the origin in field space, inhibiting the subsequent baryogenesis [97]. For instance, thermal effects will aspire to achieve a configuration of thermal equilibrium with vanishing condensate vev through Yukawa and gauge interactions, leading to a potential for any flat directions of the form $V(\psi, \bar{\psi}) \sim T^2|\psi|^2$. Furthermore, soft masses of the order of the electroweak scale give rise to contributions to the potential of the order $V(\psi, \bar{\psi}) \sim m^2_{soft}|\psi|^2$. Even more important are the Hubble induced masses of the form $V(\psi, \bar{\psi}) \sim H^2|\psi|^2$, which for the simplest possible Kähler potentials corresponding to flat field space geometry, rapidly and classically evolve any initially displaced flat direction vev to the origin in field space and efficiently prohibit the development of a non-vanishing condensate.

Nevertheless, in the formative paper [97], Dine, Randall and Thomas demonstrated that by adjusting the numerical coefficients of certain non-renormalizable operators in the Kähler potential, the sign of the Hubble induced mass-squared can be changed, thus giving rise to a tachyonic contribution to the total mass of the condensate at the origin in field space. For a non-vanishing initial vev of the flat direction at the beginning of inflation, the thermal interactions freeze out[1], thus removing the thermal contribution to the scalar potential. Moreover, since for most models $O(m^2_{soft}) \approx m^2_{EW} \ll H^2$ during inflation, the soft terms are negligible in comparison to the Hubble induced contribution. The resulting tachyonic mass of the Affleck-Dine field causes a prompt development of a significant condensate vev. The flat direction eventually settles down, stabilized by contributions to the potential arising from Planck-suppressed non-renormalizable operators in the superpotential, or, in the absence of a superpotential for the flat direction to all orders, by non-renormalizable operators in the Kähler potential.

To be explicit, a field lifted by operators of dimension $n \geq 4$ in the superpotential,

$$W \supset \frac{\lambda}{n}\frac{\Psi^n}{M_{Pl}^{n-3}}\,, \tag{3.2}$$

will have a scalar potential in which the dominant contributions during F-term inflation are given by[2]:

$$V(\psi,\bar{\psi}) = -c_I H^2|\psi|^2 + \left(a\frac{\lambda H\psi^n}{nM_{Pl}^{n-3}} + c.c.\right) + |\lambda|^2\frac{|\psi|^{2n-2}}{M_{Pl}^{2n-6}}\,, \tag{3.3}$$

[1]More precisely, the freeze-out of thermal interactions requires the initial vev of the scalar component of the flat direction to be $\psi_{in} > T/g$, as discussed in [97].

[2]Here we have made the natural assumption that the cutoff scale in the holomorphic superpotential is given by M_{Pl}, since lower scales arising from string compactifications typically depend non-holomorphically on moduli.

where H is the Hubble constant, and c_I and a are constant. Apart from neglecting the thermal and the electroweak scale soft contributions to the scalar potential as discussed above, here also contributions arising from a nontrivial Kähler potential have been neglected apart from their influence on c_I. We will have more to say about this consistent approximation in §3.4.

For c_I and λ of order one, the flat direction becomes stabilized at

$$\begin{aligned} \psi_f &\simeq M_{Pl}\left[\frac{c_I}{(n-1)|\lambda|^2}\right]^{\frac{1}{2(n-2)}}\left(\frac{H}{M_{Pl}}\right)^{\frac{1}{n-2}} \\ &\approx M_{Pl}\left(\frac{H}{M_{Pl}}\right)^{\frac{1}{n-2}}, \end{aligned} \tag{3.4}$$

while the (order n) A-term of equation (3.3) fosters a nontrivial phase upon the condensate. For small c_I, quantum fluctuations will contribute to the formation of the condensate.

After inflation, the (overdamped) condensate tracks an instantaneous minimum of the potential until the effects of the electroweak scale soft terms become non-negligible. At $H \simeq m_{EW}$ the torque exerted on the condensate by the soft A-terms triggers a spiraling motion of the vev towards the origin in field space. If the condensate is charged under a global $U(1)$ baryon symmetry[3] acting like $\psi \to e^{i\alpha}\psi$ on the field, the rotation gives rise to a non-vanishing global charge density e.g,

$$q = -i(\psi^*\dot{\psi} - \psi\dot{\psi}^*)\,. \tag{3.5}$$

For small vevs, the electroweak scale A-terms are subdominant to the soft masses, and the resulting baryon number is approximately conserved. In [97],

[3]In the MSSM the relevant symmetry is the accidental $B-L$, since baryon number by itself is not invariant under non-perturbative sphaleron processes, which are in equilibrium in the early universe.

the resulting fraction of baryon number over the number of photons was estimated to

$$\frac{n_B}{n_\gamma} \sim 10^{-10}\left(\frac{T_R}{10^9\ \mathrm{GeV}}\right)\left(\frac{M_{Pl}}{m_{soft}}\right)^{\frac{n-4}{n-2}} , \tag{3.6}$$

after the decay of the condensate through thermal scattering with the cosmological plasma. Subsequent late-time entropy releases can be necessary in order to achieve a small enough baryon asymmetry. The vev of the flat direction has also other important consequences for the thermal history of the universe, as discussed in e.g. [94, 98], and references therein.

3.2 Geometric Condition

The purpose of this Chapter is to survey the correlations in the Affleck-Dine scenario, and, fortunately, the structure of $\mathcal{N} = 1$ supergravity is highly non-trivial and the condition $c_I \approx 1$ leads to a small array of correlated predictions for various couplings in the Lagrangian. A number of authors have previously discussed the conditions under which Affleck-Dine baryogenesis is attainable (see e.g. [97, 100, 101, 99]). In this section we add to this work by obtaining a very transparent representation of the geometric condition in terms of a sectional curvature on the field space manifold, which incorporates the full inflaton dependence of the flat direction mass.

The scalar potential of $\mathcal{N} = 1$ supergravity is given in terms of an effective Kähler potential K and a superpotential W as,

$$V = V_F + V_D \quad = \quad e^{K/M_{Pl}^2}\left(K^{A\bar{B}}F_A\bar{F}_{\bar{B}} - 3\frac{|W|^2}{M_{Pl}^2}\right) + \frac{1}{2}\sum_i g_i^2 D_i^2 , \tag{3.7}$$

with the F-terms $F_A = D_A W = (\partial_A + \frac{K_A}{M_{Pl}^2})W$, where A runs over all the chiral superfields Φ^A in the theory. We have included the D-term potential for Abelian gauge groups in equation (3.7), with $D_i = \phi^A q^i_A K_A + \xi_i$, where ϕ^A is the scalar component of Φ^A, q^i_A is the $U(1)_i$ charge of ϕ^A and ξ^i is the corresponding field dependent Fayet-Iliopoulos term.

The mass-term for the flat directions arising from the potential (3.7) is easily obtained from first principles by following the standard prescription for computing soft masses in supergravity [102]: The chiral fields can be separated into the set of visible sector fields C^a (including the flat directions) and the hidden sector fields X^m (including the inflaton), where, in the visible sector, the F-terms are assumed to vanish and the vevs are taken to be small compared to M_{Pl}. It is then appropriate to make a partial Taylor expansion of the Kähler potential and superpotential around the origin in the visible sector field space:

$$\begin{aligned} K &= \tilde{K} + \tilde{K}_{a\bar{b}} C^a C^{\bar{b}} + \frac{1}{2}(\tilde{K}^{(2,0)}_{ab} C^a C^b + c.c.\,) + \ldots\,, \\ W &= \tilde{W} + \frac{1}{2}\mu_{ab} C^a C^b + \frac{1}{6}\lambda_{abc} C^a C^b C^c + \ldots\,, \end{aligned} \tag{3.8}$$

where all the expansion coefficients are function of the hidden sector fields, e.g. $\tilde{K} = \tilde{K}(X, \bar{X})$, $\mu_{ab} = \mu(X)_{ab}$. Additional information about the chiral and gauge structure of the theory needs to be supplied separately; for instance in the MSSM, the only non-vanishing constant contribution to the μ-term in the superpotential allowed by gauge and R-parity invariance is $H_u H_d$. The same symmetries similarly severely restrict the allowed operators in the Kähler potential.

We will be particularly interested in the *renormalizably flat directions* of the globally supersymmetric theory, for which no gauge and R-parity invariant operator in the superpotential below order $n \geq 4$ exist. In the MSSM, the numerous gauge invariant monomials corresponding to single flat directions were listed in [103], and recently in full detail in [104].

The renormalizable superpotential gives rise to interactions between different independently flat directions, so that the presence of a vev of one field may lift many others. We will subsequently refer to a set of flat directions that remain renormalizably flat in the presence of arbitrary vevs of the other elements of the set, as a *sector* of flat directions. The different sectors can be studied by constructing gauge invariant polynomials, as discussed in [105, 106], where the $H_u L_i$ directions were explicitly constructed.

Denoting the flat directions in some sector by Ψ^a, the superpotential in this sector can be written,

$$W = \tilde{W}(X) + \frac{1}{nM_{Pl}^{n-3}} \lambda^{(n)}_{a_1 \ldots a_n}(X)\, \Psi^{a_1} \ldots \Psi^{a_n} + \ldots \tag{3.9}$$

The leading order terms in the resulting Lagrangian are easily obtained [102]: under these assumptions the holomorphic bilinears, holomorphic trilinears and fermion masses all vanish for vanishing flat direction vevs (i.e. $B_{ab} = 0$, $A_{abc} = 0$ and $m_\psi = 0$ at $\psi^a = 0$), and the mass matrix for the scalars with kinetic terms given by $\tilde{K}_{a\bar{b}}$ is given by,

$$\begin{aligned} \partial_a \bar{\partial}_{\bar{b}} V_F &= \left(e^{\tilde{K}/M_{Pl}^2} \frac{F^{\bar{n}} \bar{F}_{\bar{n}}}{M_{Pl}^2} - 2m_{3/2}^2 \right) \tilde{K}_{a\bar{b}} - e^{\tilde{K}/M_{Pl}^2} F^{\bar{m}} \bar{F}^n R_{n\bar{m}a\bar{b}} \\ &= \left(3H^2 - \frac{V_D}{M_{Pl}^2} + m_{3/2}^2 \right) \tilde{K}_{a\bar{b}} - e^{\tilde{K}/M_{Pl}^2} F^{\bar{m}} \bar{F}^n R_{n\bar{m}a\bar{b}} \,, \end{aligned} \tag{3.10}$$

where we have used $V_F + V_D = V \simeq 3H^2 M_{Pl}^2$, and introduced the gravitino mass $m_{3/2}^2 = e^{\tilde{K}/M_{Pl}^2} |\frac{\tilde{W}}{M_{Pl}^2}|^2$, and the field space curvature $R^{\bar{m}}_{\ \bar{n}a\bar{b}} = \partial_a \tilde{\Gamma}^{\bar{m}}_{\bar{n}\bar{b}}$. Furthermore, here $\tilde{K}^{m\bar{n}} \bar{F}_{\bar{n}} = \bar{F}^m$. For a non-vanishing $U(1)$ D-term potential, this contribution to the scalar mass matrix should be complemented with the contribution from V_D, as discussed in e.g. [100, 107].

The n-th order A-term of (3.3) can similarly be worked out from V_F, and includes contributions from the superpotential as well as from the n-th order terms in the partial Taylor expansion of the Kähler potential, i.e. $\tilde{K}^{(n,0)}(X,\bar{X})\psi^n$. We will have more to say about this coupling in §3.4.

3.2.1 Holomorphic Bisectional Curvature

Specializing to the particularly illuminating case of a single hidden sector field, ϕ, supporting the energy density of the early universe and driving inflation (hence, the inflaton) and a single flat direction, ψ, while momentarily specializing to the case $V_D = 0$, we find that the canonically normalized mass for the flat direction at the origin in field space is given by,

$$m^2_{\psi\bar{\psi}} = 3H^2\left(\left[1+\frac{1}{3}\left(\frac{m_{3/2}}{H}\right)^2\right]-\left[1+\left(\frac{m_{3/2}}{H}\right)^2\right]M^2_{Pl}\tilde{K}^{\phi\bar{\phi}}\tilde{K}^{\psi\bar{\psi}}R_{\phi\bar{\phi}\psi\bar{\psi}}\right). \quad (3.11)$$

The gravitino mass, $m_{3/2}$, may during inflation take on values as large as H without fine-tuning, and is not necessarily related to the late time gravitino mass. The expression,

$$B[\phi,\psi] = -M^2_{Pl}\tilde{K}^{\phi\bar{\phi}}\tilde{K}^{\psi\bar{\psi}}R_{\phi\bar{\phi}\psi\bar{\psi}}\,, \quad (3.12)$$

is the (dimensionless) *holomorphic bisectional curvature* between the holomorphic curves — or, equivalently, between the real planes invariant under the complex structure rotations — defined by ϕ and ψ respectively, evaluated at $\psi = 0$. The holomorphic bisectional curvature, first introduced in [108], is one of the most natural concepts of curvature on a Kähler manifold and has recently proven to be a very useful concept in relating local quantities in complex geometry to global theorems, see e.g. [109].

Of direct relevance for Affleck-Dine baryogenesis is that the the parameter c_I in equation (3.3) is given by

$$c_I = -3\left(1 + B[\phi,\psi] + \left(\frac{m_{3/2}}{H}\right)^2 \left(\frac{1}{3} + B[\phi,\psi]\right)\right). \tag{3.13}$$

Since during inflation, H necessarily remains approximately constant and so does $m_{3/2}$ for most reasonable models, the functional dependence of c_I on the inflaton vev ϕ is determined solely by $B[\phi,\psi]$.

Thus for F-term inflation, tachyonic masses at the origin in field space require a negative holomorphic bisectional curvature. In particular, in the interesting case when $m_{3/2} \simeq H$ during inflation, Affleck-Dine baryogenesis is conditioned on $B[\phi,\psi] \lesssim -\frac{2}{3}$, while for $m_{3/2} \ll H$ during inflation, the condition sharpens slightly to $B(\phi,\psi) < -1$.

To illustrate the utility of equation (3.13), let us consider the canonical example of Affleck-Dine baryogenesis in F-term inflation with $m_{3/2}^2 \simeq H^2$, previously discussed in e.g. [97, 100], in which the transition of the condensate is triggered by a non-renormalizable Kähler potential of the form

$$K = |\phi|^2 + |\psi|^2 + \frac{\beta}{M_{Pl}^2}|\phi|^2|\psi|^2. \tag{3.14}$$

Since the Hubble parameter is approximately constant during inflation, the non-trivial inflaton dependence of c_I comes entirely from $B[\phi,\psi]$, which in this case is given by,

$$B[\phi,\psi] = -\left(\frac{\beta}{(1+\beta|\tilde{\phi}|^2)^2}\right), \tag{3.15}$$

where $\tilde{\phi} = \frac{\phi}{M_{Pl}}$. Clearly, the condition $B(\phi, \psi)|_{\phi=0} < -\frac{2}{3}$ translates into $\beta > \frac{2}{3}$, and equation (3.13) becomes,

$$c_I \;\; = \;\; -4\left(1 + \frac{3}{2}B[\phi, \psi]\right) = -4\left(1 - \frac{3\beta}{2(1+\beta|\tilde{\phi}|^2)^2}\right). \qquad (3.16)$$

This illustrates a point that we will return to in §3.3: c_I is already classically a nontrivial function of the inflaton, and keeping this function sufficiently negative for the condensate to remain displaced during inflation and until $H^2 \approx m^2_{soft}$ results in a nontrivial condition on the Kähler potential. From the simple example (3.16) it follows that for a field excursion of the inflaton larger than $\tilde{\phi} = \sqrt{3/8}$, there is no value of the constant β that can keep the condensate from transitioning back to the origin [100]. A reincarnation of this point will become particularly important for the discussion of large-field inflation in §3.3.3.

Let us comment on the generalization of the above considerations to multiple flat directions: clearly the negative contribution to the masses comes from the Riemann curvature $R_{a\bar{b}\phi\bar{\phi}}$, which certainly is not necessarily proportional to the metric on the moduli space $K_{a\bar{b}}$. It follows that the matrix $(c_I)_{a\bar{b}}$ is in general not universal or even diagonal.

Let us end this section by commenting on the case when a non-vanishing D-term potential is included, and the total mass for the flat direction is given by,

$$m^2_{\psi\bar{\psi}} \;\; = \;\; m^2_{3/2}(1 + 3B[\phi, \psi]) + 3H^2\Big(1 - \frac{V_D}{V}\Big)(1 + B[\phi, \psi]) + \tilde{K}^{\psi\bar{\psi}}\partial^2_{\psi\bar{\psi}}V_D\,. \quad (3.17)$$

If the Affleck-Dine field is charged under the anomalous $U(1)$ in V_D, the last term of (3.17) can contribute with a mass-squared of order $\frac{V_D}{M^2_{Pl}}$ of either sign.

If the Affleck-Dine field is uncharged under the anomalous $U(1)$, then the contribution to the mass matrix from the D-term potential, i.e. $\tilde{K}^{\psi\bar{\psi}}\partial^2_{\psi\bar{\psi}}V_D$, will arise only at loop-order. The relative magnitude of the D-term potential and the total potential affects the F-term contribution to the flat direction mass, which for a single flat direction can be written,

$$c_I^{(F)} = -3\left(\frac{4}{3} - \frac{V_D}{V} + \left(2 - \frac{V_D}{V}\right) B[\phi,\psi]\right) . \tag{3.18}$$

In this case, still assuming $m_{3/2}^2 \simeq H^2$ and now specializing to the case $\partial^2_{\psi\bar{\psi}} V_D \ll H^2$, the origin in field space become unstable for the flat direction for

$$B[\phi,\psi] < -\left(\frac{4 - 3(\frac{V_D}{V})}{6 - 3(\frac{V_D}{V})}\right) , \tag{3.19}$$

which for $\frac{V_D}{V} < \frac{4}{3}$ bounds $B[\phi,\psi]$ from above by some negative number. In the window $\frac{V_D}{V} \in (\frac{4}{3}, 2)$, Affleck-Dine baryogenesis may proceed with a positive holomorphic bisectional curvature. Examples of the Affleck-Dine mechanism in this range can be constructed — at least in field theory — by lifting an AdS minimum of V_F by a D-term potential, to the positive energy density of the inflationary epoch. Finally, we note that for a vanishing expectation value of the F-term potential, i.e. $V_F = 0$, the mass of the flat direction is $(1 + 3B[\phi,\psi])m_{3/2}^2 + \tilde{K}^{\psi\bar{\psi}}\partial^2_{\psi\bar{\psi}} V_D$, where the first term arises from nontrivial derivatives on the F-term potential that only vanish in the $W \to 0$ limit.

3.2.2 String Theory Examples

We have shown that the holomorphic bisectional curvature determines the mass of the flat direction in supergravity and that a successful Affleck-Dine baryogenesis places certain upper bounds on $B[\phi,\psi]$. Since the connection between the inflaton and the standard model provided by $B[\phi,\psi]$ arises from Planck-suppressed operators in the Kähler potential, the mechanism provides a window of ultraviolet sensitivity to string theory, in which such operators, at least in principle, can be computed. In this section we will illustrate how this can be done through examples of inflation in string compactifications, demonstrating that $B[\phi, \psi]$ is not an arbitrary function of the inflaton vev in some well-defined string constructions, and that Affleck-Dine baryogenesis in fact can be shown to be impossible in broad classes of scenarios. Consequently, if it can be established that the Affleck-Dine mechanism is indeed responsible for the observed baryon asymmetry of the universe, this would serve as a nontrivial selection criterion for string theory realizations of inflation and the standard model.

First, let us review the feasibility of Affleck-Dine baryogenesis in the volume modulus inflation of [110], in the Large Volume Scenario, [27], as previously discussed in [99]. In this model, the large volume modulus with scalar component τ_b, is displaced far from its final, approximately Minkowski, vacuum, and during inflation $m_{3/2}^2 \gg m_{EW}^2$. Assuming that the visible sector is localized in the internal dimensions, and that the visible sector fields have a diagonal Kähler metric, $\tilde{K}_{a\bar{b}} = \tilde{K}^{(a)}\delta_{a\bar{b}}$, it can be argued that the physical Yukawa couplings should be independent of the overall volume [65]. Since the holomorphic Yukawa couplings are independent of the Kähler moduli to all orders in perturbation theory, it follows from the supergravity formula for the physical Yukawa couplings,

$$Y_{abc}^{\text{Phys.}} = e^{\hat{K}/2M_{Pl}^2} \frac{Y_{abc}^{\text{Hol.}}}{\sqrt{\tilde{K}^{(a)}\tilde{K}^{(b)}\tilde{K}^{(c)}}}\,, \tag{3.20}$$

that the overall volume moduli dependence of the kinetic terms of the visible sector fields is related to $\hat{K}$ by

$$\tilde{K}^{(a)} = e^{\hat{K}/3M_{Pl}^2}\tilde{\kappa}^{(a)}\,, \tag{3.21}$$

where $\kappa^{(a)}$ is independent of the overall volume modulus. This determines the coupling between the inflationary and the visible sector to be,

$$B[\phi = \tau_b, \psi] = -\frac{1}{3}\,, \tag{3.22}$$

which is identical to the holomorphic bisectional curvature for no-scale Kähler potentials. It follows from equation (3.13) that for F-term inflation, the contribution proportional to $m_{3/2}^2$ famously drops out, while the Hubble induced mass gives,

$$c_I = -4 - 6B[\phi, \psi] = -2\,. \tag{3.23}$$

It follows that Affleck-Dine baryogenesis is not possible for volume modulus inflation, or for any F-term inflationary model based on a no-scale Kähler potential [99].

A second, slightly more nontrivial example that to our knowledge previously has not been discussed in the literature is Affleck-Dine baryogenesis in the context of brane inflation in the KKLT scenario of moduli stabilization [26, 57]. At the level of the four-dimensional effective theory, this system can be modeled by supplementing the supergravity F-term potential by an explicitly supersymmetry breaking uplift potential,

$$V_{\text{tot.}} = V_F + V_{up}\,, \tag{3.24}$$

and the masses in the visible sector are given by equation (3.17), after replacing V_D by V_{up}. There exists an interesting class of models in which warping has been argued to ensure a sequestered form of the Kähler potential [111, 112],

$$K = -3M_{Pl}^2 \ln\left(-\frac{1}{3}(f_{vis.} + f_{hid.})\right)\,, \tag{3.25}$$

we again find that,

$$B[\phi, \psi] = -\frac{1}{3}\,. \tag{3.26}$$

In KKLT, the vev of the F-term potential during inflation is approximately given by $V_F \simeq -3m_{3/2}^2 M_{Pl}^2$, and the Hubble constant during inflation can not exceed the gravitino mass [113]. Writing $m_{3/2}^2 = (1+\beta)H^2$, it follows that $\frac{V_{up}}{V} = 2+\beta$, for some constant $\beta > 0$. Using equation (3.17), we find that

$$c_I^{(F)} = 2 + 2\beta\,. \tag{3.27}$$

However, the uplift potential depends on the visible sector fields, and contributes to c_I with [2],

$$c_I^{(up)} = -\frac{2}{3}\frac{V_{up}}{M_{Pl}^2 H^2} = -4 - 2\beta\,. \tag{3.28}$$

In conclusion, we find that

$$c_I = -2\,, \tag{3.29}$$

from which it follows that Affleck-Dine baryogenesis is not possible in brane inflation in KKLT with a sequestered visible sector.

The Affleck-Dine mechanism can be embedded in some string models, see e.g. [100], in which it was noticed that the tree-level Kähler potential in orbifold theories gives rise to, in our notation, a constant holomorphic bisectional curvature,

$$B[T, \psi_i] = \frac{n_i}{3} \tag{3.30}$$

where T is the overall volume modulus and the candidate inflaton to boot, and n_i, being an integer in the interval $[-1, -5]$, is the modular weight of the chiral field ψ_i. Further examples of the viability of the Affleck-Dine mechanism in effective theories coming from string theory can be found in [99].

The difficulty in obtaining positive c_I in string theory models of inflation can be traced back to the fact that though the underlying no-scale symmetry is broken in stabilized compactifications, it can still importantly influence the Planck suppressed operators determining the coupling between the visible sector and the inflaton. While in Minkowski space the no-scale structure cause vanishing scalar masses, in the quasi-de Sitter space relevant for inflation on the other hand, the cancellation is only partial [114], and the resulting mass-squared is always positive.

3.3 Correlated Predictions: *Backreaction on the inflaton*

In §3.2 we have discussed how the necessary condition for Affleck-Dine baryogenesis can conveniently be written in terms of $B[\phi, \psi]$ in the single flat direction case, and how this bisectional curvature captures the functional behavior of the mass of the flat direction at the origin in field space as a function of the inflaton vev. In §3.4 we will discuss how $B[\phi, \psi]$ appears in other places in the supergravity Lagrangian, and thus gives rise to correlated predictions of the Affleck-Dine scenario. In this section, however, we focus on a particular cosmological consequence of the Affleck-Dine scenario which potentially can severely constrain the mechanism, namely the what can be thought of as the 'backreaction' of the transitioning flat direction on the inflaton. Two questions are especially pertinent for this analysis: First, if the Affleck-Dine field transitions from some initial vev to ψ_f of equation (3.4) during the circa ten e-folds when the cosmological scales left the horizon, what are the resulting cosmological signatures? Second, to what extent can it be natural to expect a flat direction transitioning during this period?

The first question closely connects to a large body of work on the cosmological effects of features in the inflaton potential, and, in particular, it partially overlaps with the "multiple inflation" scenario of [115]. More generally, a transitioning field gives rise to a short period of multi-field, non-slow roll inflation, and a complete analysis of this period requires an extension of the works of e.g. [116, 117, 118] to apply also for swiftly turning field trajectories. A full treatment of this multi-field system is a very interesting future direction, here, however, we take a first step towards understanding what cosmological constraints can be placed on these systems by comparing it to known constraints on simpler single-field systems. This simplified analysis can be thought of as modeling the behavior of the *longitudinal* component of the multi-field system, while neglecting the effects of fluctuations transverse to the evolution of the field. We therefore expect that the resulting constraints from the multiple-field analysis to be even more severe than the corresponding parameter bounds obtained below from the single-field analogy, which motivates the study of these bounds in the simplified system.

Concretely, by mapping the effects of a transitioning field to a *step* in an effective, single-field inflaton potential, and by using bounds on the height and width of the step from [119] (see also [120, 121, 122, 123, 124, 125, 128]) extracted from the seven year WMAP data of the CMB temperature anisotropies, we find in §3.3.2 that the level, n, at which the flat direction is lifted in the superpotential and the value of the function c_I, can both be severely constrained in certain versions of the scenario.

In §3.3.3, the question of naturalness is addressed for both small-field and large-field models of inflation, for both thermal and non-thermal initial conditions. For small-field inflation with initially thermally trapped flat directions, we find that observing traces of a transitioning flat direction through the CMB can be perfectly natural during the first 20 to 30 e-folds of inflation, and modest fine-tuning can prolong this period substantially. For large-field inflation, transitions might not only be natural, but also abundant during inflation. The near scale-invariance of the CMB on the other hand severely constrains any transitions occurring at random places during inflation, which forces the Kähler potential of the large-field model to have a very special form.

For earlier discussions of the possibility of constraining the initial configuration of the flat directions in the context of Affleck-Dine baryogenesis using scale invariance, see [129].

3.3.1 Constraining the transitioning field

While the analysis of the temperature anisotropies of the cosmic microwave background has given substantial evidence that a period of inflation occurred in the early universe, there are still many open questions regarding the exact nature of this period of accelerated expansion. For example, in theoretical models of slow-roll inflation, certain flatness conditions are enforced on the inflaton potential, *features* in the inflaton potential however, can be admissible or even favored by the current WMAP data [121, 119]. In fact, recent analysis of the temperature anisotropies measured by WMAP [130], QUaD [131] and ACBAR [132] find an improved fit for inflaton potentials with a small step located at a specific location of the inflaton potential [119], consistent with earlier analysis [121, 122, 128]. The size of the step is constrained from data to be no larger than around .1% in large classes of inflationary models. While these results are intriguing, future observations of the E-mode polarization of the CMB spectrum as well as improved bounds on the bispectrum will be able to determine the cosmological significance of these features [123, 134, 135, 133].

Historically, transitioning flat directions have served as one of the main motivations for the study of localized features in the inflaton potential. Here, we discuss the effects of transitioning flat directions in the context of Affleck-Dine baryogenesis by mapping the system onto a simplified single-field model with a step in the potential, for which the cosmological constraints from the CMB spectrum are known. The subset of models of Affleck-Dine baryogenesis that can be constrained this way partially overlap with the scenario of multiple inflation of [115], in which an initially thermally trapped supersymmetric flat direction transitions to a significant vev, and while doing so 'backreacts' on the inflaton potential. However, while [115] and the subsequent works [124, 125] focus on a specific small-field inflationary model with a rather small Hubble constant ($H \simeq 10^{-8} M_{Pl}$), it is certainly interesting to generalize these considerations to broader classes of inflationary models as well as considering more general initial conditions, as we will do in §3.3.3.

To motivate the mapping of the multiple-field system to a single-field system with a localized step, we consider the Affleck-Dine potential (3.3) supplemented with an inflaton potential $V_0(\phi)$, supporting small-field slow-roll inflation,

$$V(\phi, \psi) \quad = \quad V_0(\phi) - c_I(\phi) H_I^2 \psi^2 + |\lambda|^2 \frac{\psi^{2n-2}}{M_{Pl}^{2n-6}}\,, \tag{3.31}$$

where we have neglected the order-n A-term as well as the phase of the condensate. The equations of motion of the system are given by,

$$\begin{cases} \ddot{\psi}(t) + 3H(t)\dot{\psi}(t) + \frac{\partial V(\phi,\psi)}{\partial \psi} = 0 \\ \ddot{\phi}(t) + 3H(t)\dot{\phi}(t) + \frac{\partial V(\phi,\psi)}{\partial \phi} = 0 \\ H(t)^2 = \frac{1}{3M_{Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\psi}^2 + V(\phi,\psi)\right)\,. \end{cases} \tag{3.32}$$

While a transitioning flat direction affects the cosmological perturbations for a variety of initial conditions, for concreteness let us consider the case when $\psi_i \simeq H$ at some time $t = 0$ when the transition begins, and postpone the discussion of initial conditions to §3.3.3. In this case, an analytic solution for $\psi(t)$ is readily obtained at early times, i.e. for $\psi \lesssim \psi_f$,

$$\psi(t) = \psi_i e^{\frac{3Ht}{2}\left(\sqrt{1+\frac{8}{9}c_I}-1\right)}, \tag{3.33}$$

where we have assumed that the transition is prompt so that H and $c_I(\phi)$ are approximately constant during the transition. During the transition, the two-field system (ϕ, ψ) evolves from a 'ridge' of the potential to settle down in the 'valley' at ψ_f, much like a gentle version of the waterfall transition common in models of hybrid inflation (where $c_I \gg 1$, and the transitions terminates the inflationary era). Longitudinal fluctuations of the fields along the instantaneous tangent vector of the field trajectory give rise to curvature perturbations, while fluctuations orthogonal to the field trajectory result in entropy perturbations, as discussed in the case of slow-roll inflation in [116, 126]. Futhermore, for c_I of order one and with an unsuppressed dependence on ϕ, a large-field version of this system has been analyzed in [127], where it was noticed that quantum back-reaction can typically not be neglected. Extending the two-field analysis to the system of equations (3.32) is beyond the scope of this thesis. Fortunately however, much can be learned by mapping the two-field system onto a single-field system with a potential with a small step, corresponding to the step induced by the transition of the flat direction along the *longitudinal* motion in the (ϕ, ψ) field space. In this sense, the transitioning flat direction 'backreacts' on an effective, longitudinal, single-field inflaton potential to induce a step in it.

To estimate the steepness of the step in the single-field potential, we note that the duration, $t_\star$, of the transition from $\psi_i = H$ to $\psi_f > \psi_i$ of (3.4), can be estimated as,

$$t_\star = \frac{2}{3kH}\ln\left(\frac{\psi_f}{\psi_i}\right) \simeq \frac{1}{H}\left(\frac{n-3}{n-2}\right)\frac{2}{3k}\ln\left(\frac{M_{Pl}}{H}\right), \tag{3.34}$$

which provides a lower bound on — and a good approximation to — the actual transition time. Here we have abbreviated $k = \sqrt{1+\frac{8}{9}c_I} - 1$. Expressed in terms of the longitudinal velocity $|\dot\phi_\|| \equiv \sqrt{\dot\phi^2+\dot\psi^2}$, the slow-roll parameters,

$$\epsilon = -\frac{\dot H}{H^2}, \tag{3.35}$$

$$\eta_\| = \frac{\ddot\phi_\|}{H|\dot\phi_\||}, \tag{3.36}$$

deviate significantly from their initial, slow-roll, values during the transition. On the other hand, the speed in the ϕ-direction does not change abruptly during the transition and can be approximated by its initial slow-roll value[4] $\dot\phi \simeq -\sqrt{2\epsilon_V(\phi_i)}HM_{Pl}$, where $\epsilon_V = \frac{M_{Pl}^2}{2}\left(\frac{\partial_\phi V}{V}\right)^2$ and ϕ_i denotes the inflaton vev just before the transition. The change in the inflaton vev during the transition can then be estimated as,

$$\Delta\phi \simeq -\sqrt{2\epsilon_V(\phi_i)}\,\frac{2}{3k}\ln\left(\frac{\psi_f}{\psi_i}\right) M_{Pl} \simeq -\frac{\sqrt{8/3}}{2.7^2k}\cdot 10^4\left(\frac{n-3}{n-2}\right)\ln\left(\frac{M_{Pl}}{H}\right) H, \tag{3.37}$$

assuming that the transition happens reasonably close to the time when cosmological scales left the horizon, so that ϵ_V is related to the Hubble scale by the COBE normalization,

$$\frac{V^{1/4}}{\epsilon^{1/4}} = 2.7\cdot 10^{-2}M_{Pl}. \tag{3.38}$$

The size of the step can readily be obtained by evaluating $\Delta V = V(\phi,\psi_f) - V(\phi,\psi_i)$, for which we find,

$$\frac{\Delta V}{V_0} \simeq -\left(\frac{n-2}{n-1}\right)\frac{c_I}{3}\left[\frac{c_I}{(n-1)|\lambda|^2}\right]^{\frac{1}{n-2}}\left(\frac{H}{M_{Pl}}\right)^{\frac{2}{n-2}}. \tag{3.39}$$

[4]This approximation breaks down if $c_I(\phi)$ contains a linear term in ϕ with an order one coefficient, as we discuss below.

Thus, we propose to model the two-field model as a single-field model with a step, parametrized as,

$$V(\varphi) = V_0(\varphi)\left(1 - c_f \tanh\left(\frac{\varphi - \varphi_f}{d_f}\right)\right), \tag{3.40}$$

with size and width approximately given by

$$c_f = \left(\frac{n-2}{n-1}\right)\frac{c_I}{3}\left[\frac{c_I}{(n-1)|\lambda|^2}\right]^{\frac{1}{n-2}}\left(\frac{H}{M_{Pl}}\right)^{\frac{2}{n-2}}, \tag{3.41}$$

$$d_f \simeq \frac{\phi_{\|}}{4M_{Pl}} = \frac{1}{4M_{Pl}}\int_0^{t_\star} dt\ \sqrt{\dot{\psi}^2 + \dot{\phi}^2} \tag{3.42}$$

where $\phi_{\|}$ denotes the path length from (ϕ_i, ψ_i) to (ϕ_f, ψ_f) as measured with the approximately Euclidean field space metric, and is easily evaluated using the approximate solutions for ψ and ϕ. The single-field model thus corresponds to the longitudinal coordinate along the field trajectory in the (ϕ, ψ)-plane.

In §3.3.2, we perform a more detailed numerical analysis of an example of a transitioning flat direction during inflation, and discuss how the bounds on the parameters c_f and d_f can be interpreted as bounds on n and c_I for a given inflationary model.

Consequences of an inflaton dependent c_I

While the mapping of the transitioning system to an inflaton potential with a step is well motivated and serves to give a rough idea of what constraints can be imposed on the system, the multi-field system contains a rich variety of physics, that can not all be captured by a single-field inflaton potential with a step [136, 133].

The neglect of multi-field effects is most severe when the fields evolve through a sharp turn in field space, as happens when the ψ undergo damped oscillations to settle down at ψ_f,

$$\psi(t) = \psi_f \left(1 + Ae^{-\frac{3Ht}{2}} \cos(\omega t + \vartheta)\right) , \tag{3.43}$$

with $\omega^2 = \left(4c_I^{(0)}(n-2) - \frac{9}{4}\right) H^2$, and for integration constants A and ϑ. Computing the perturbations around this background solutions is an interesting future problem.

Futhermore, the function $c_I(\phi)$ is typically not a constant, as we have argued in §3.2.1, and can be Taylor expanded in $\tilde{\phi} = \frac{\phi}{M_{Pl}}$,

$$c_I(\phi) = c_I^{(0)} + c_I^{(1)}\tilde{\phi} + \frac{1}{2}c_I^{(2)}\tilde{\phi}^2 + \dots \tag{3.44}$$

Once multiplied by the vacuum expectation value of a flat direction, the inflaton dependence in equation (3.44) leads to corrections to the inflaton potential in the single-field model, that can not be captured by a step in an otherwise unperturbed inflaton potential. The importance of these corrections can be estimated by considering the effects of a flat direction transitioning during observable inflation, as the unperturbed inflaton potential satisfies the COBE normalization (3.38). The unperturbed inflaton potential can be Taylor expanded around ϕ_0, such that

$$V_0(\phi) = 3H^2 M_{Pl}^2 \left(1 - a_1 \Delta\tilde{\phi} + \frac{a_2}{2}\left(\Delta\tilde{\phi}\right)^2 + \dots\right) , \tag{3.45}$$

where $\Delta\tilde{\phi} = \frac{\phi - \phi_0}{M_{Pl}}$. This expansion is good for all small field models, in which $\Delta\tilde{\phi} \ll 1$, as well as for some large-field models, e.g. those with monomial potentials, since in that case the structure of the expansion coefficients a_i ensure that the true expansion parameter is $\frac{\Delta\phi}{\phi_0}$, which is smaller than unity before the end of inflation. The slow-roll parameters at ϕ_0 are given by, $\epsilon_V^{(0)} = \frac{1}{2}a_1^2$ and $\eta_V^{(0)} = a_2$.

Imposing the COBE normalization of equation (3.38) on the inflaton potential (3.45) at ϕ_0 gives that,

$$a_1 = \frac{\sqrt{6} \cdot 10^4}{2.7^2} \left(\frac{H}{M_{Pl}} \right) . \tag{3.46}$$

A flat direction transitioning just as the inflaton passes ϕ_0 changes the inflaton potential by

$$\Delta V(\phi) = -\left(c_I^{(0)} + c_I^{(1)} \Delta\tilde{\phi} + \quad \frac{1}{2} c_I^{(2)} \Delta\tilde{\phi}^2 + \ldots \right) H^2 \left(\frac{H}{M_{Pl}} \right)^{\frac{2}{n-2}} . \tag{3.47}$$

The slow-roll parameters change correspodingly: in terms of the Lagrangian cofficients a_1 and a_2, we find that

$$\begin{aligned} \frac{\Delta a_1}{a_1} &= \frac{2.7^2}{3\sqrt{6}} c_I^{(1)} 10^{-4} \left(\frac{M_{Pl}}{H} \right)^{\frac{n-4}{n-2}} , & (3.48) \\ \frac{\Delta a_2}{a_2} &= \frac{c_I^{(2)}}{3\eta_V^{(0)}} \left(\frac{H}{M_{Pl}} \right)^{\frac{2}{n-2}} . & (3.49) \end{aligned}$$

From (3.48), we find that the condensate vev changes ϵ_V by an $O(1)$ factor if

$$n \geq 2 \left(\frac{\log\left(|c_I^{(1)}| \left(\frac{M_{Pl}}{H} \right)^2 \right) - 4}{\log\left(|c_I^{(1)}| \left(\frac{M_{Pl}}{H} \right) \right) - 4} \right) , \tag{3.50}$$

while η_V is sensitive for transitioning flat directions with an n greater than

$$n \geq 2 \left(1 + \frac{\log\left(\frac{M_{Pl}}{H} \right)}{\log\left| \frac{c_I^{(2)}}{\eta_V^{(0)}} \right|} \right) . \tag{3.51}$$

For example, the large-field model with a quadratic potential $V_0(\phi) = \frac{1}{2} m^2 \phi^2 = \frac{1}{2} m^2 \phi_0^2 \left(1 - \frac{\Delta\phi}{\phi_0} \right)^2$ and a Hubble scale close to $10^{-4} M_{Pl}$ is rather insensitive to corrections in the the tilt (ϵ_V) but is sensitive to corrections in the curvature (η) for condensates transitioning with $n \gtrsim 6$. Once the condensate has formed, it will in general not be possible to reassemble the inflaton potential to a monomial potential again.

Backreaction on moduli fields

Known ultraviolet-complete models of inflation typically have a spectrum that contains comparatively light particles with masses around H. It is therefore important to understand if a transitioning Affleck-Dine field can affect the moduli in a way that could possibly give rise to additional observational signatures.

In order to be concrete, we will discuss this question in a particular example based on the "Kähler moduli inflation" in the Large Volume Scenario, see [137]. In this case, the mass of the lightest (and largest) Kähler modulus is given by [138],

$$m^2 \simeq \frac{1}{\ln \mathcal{V}} H^2 \simeq \frac{1}{15} H^2 \,, \tag{3.52}$$

where we have used that the volume is large in string units, $\mathcal{V} \approx 10^5$.

From the special form of the Kähler potential determined by equation (3.21), the coupling between $\psi^\dagger\psi$ and the canonically normalized volume modulus $\Phi = \sqrt{\frac{2}{3}} \ln \mathcal{V}$, includes the term

$$V \supset \psi^\dagger\psi \frac{1}{\mathcal{V}^{2/3}} \frac{V_F}{M_{Pl}^2} \approx 3H^2 M_{Pl}^2 \left(\frac{\psi_f}{M_{Pl}}\right)^2 e^{-\sqrt{2/3}\Phi} \,. \tag{3.53}$$

If the modulus Φ is stabilized at the vev Φ_0 with mass m during inflation, the transitioning flat directions induce a shift in the modulus which to leading order in $\delta\Phi/M_{Pl}$ is given by,

$$\delta\Phi = \sqrt{6}\, \frac{H^2}{\tilde{m}^2} \left(\frac{H}{M_{Pl}}\right)^{\frac{2}{n-2}} \frac{1}{\mathcal{V}^{2/3}} M_{Pl} \,, \tag{3.54}$$

where we have defined $\tilde{m}^2 = m^2 + 2H^2 (\frac{H}{M_{Pl}})^{\frac{2}{n-2}} \frac{1}{\mathcal{V}^{2/3}}$. Thus for $m \approx H$ during inflation, the shift in the vev of the modulus due to the displaced flat direction is small with respect to M_{Pl}, and will not give rise to an observable signature. In the Large Volume Scenario, the volume suppression of equation (3.54) makes the shift truly negligible. In the broader class of models in which the moduli couples to the flat direction only through the Kähler potential, the suppression in powers of $(\frac{H}{M_{Pl}})$ is generic, and the resulting shifts in moduli vevs are small.

3.3.2 Numerical Analysis

Let us now discuss a specific toy model of an Affleck-Dine transition during inflation, in order to demonstrate in detail how bounds on the size and width of the step in the potential in the single-field model can provide interesting constraints the parameters n and c_I for flat directions transitioning during observable inflation.

Specifically, we consider a two-field potential for the real, scalar, inflaton ϕ, and the radial component of a flat direction, represented by the real scalar field ψ, given by

$$V(\phi,\psi) \quad = \quad V_I(\phi) - c_I^{(0)} H_I^2 \psi^2 - c_I^{(1)} \frac{H_I^2}{M_{Pl}}\, \phi\, \psi^2 + |\lambda|^2 \frac{\psi^{2n-2}}{M_{Pl}^{2n-6}}\,, \tag{3.55}$$

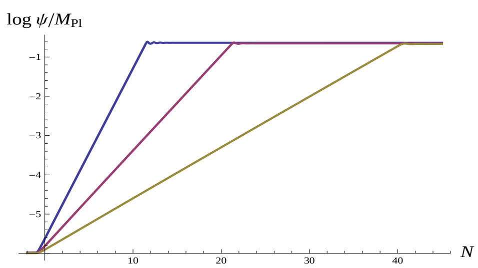
where $V_I(\phi)$ is the unperturbed, initial, inflaton potential, H_I is a constant approximately equal to the value of the Hubble parameter during inflation, and we have included a linear term in the Taylor expansion of $c_I(\phi)$. Considering a small-field model of inflation, for which the inflaton potential, at least locally, can be Taylor expanded as,

$$V_I(\phi) = V_0 + V_1\phi + V_2\phi^2\,, \tag{3.56}$$

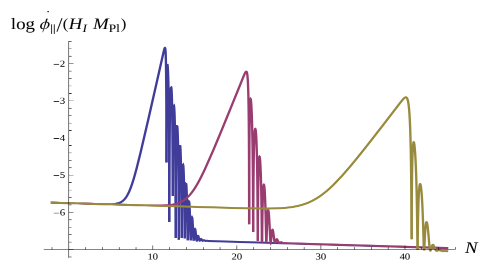
we investigate the effects of varying $c_I^{(0)}$ and n for fixed λ and for fixed parameters of $V_I(\phi)$. We specialize to the particular case of $H_I \simeq 10^{-6} M_{Pl}$, and

$\eta_V \simeq .67 \cdot 10^{-2}$ initially, while imposing the COBE normalization on the inflaton potential just before the transition.

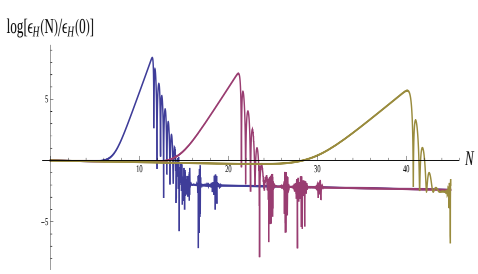
For an initial vev of the flat direction, $\psi_i = H_I$, the initial conditions for the inflaton were chosen so that the system starts out in slow-roll close to the origin in field space. The naturalness of these assumptions will be discussed in §3.3.3. As ψ condenses, the two-field system slants from the 'ridge' in the potential at $\psi = 0$, down to the 'valley' at $\psi = \psi_f$, where it settles down during a short period of damped oscillations, while simultaneously slowly rolling in the ϕ-direction.



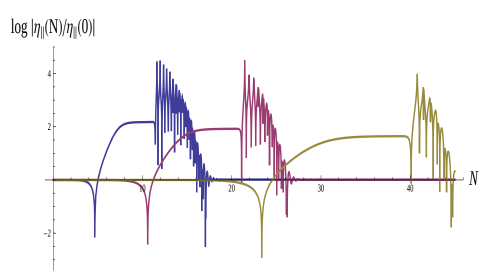
(a) The evolution of the condensate vev.



(b) The evolution of the longitudinal speed.



(c) The evolution of the slow-roll parameter ϵ_H of equation (3.35).



(d) The evolution of the slow-roll parameter $\eta_{\|}$ of equation (3.36).

Figure 3.1: The evolution of the condensate vev, the longitudinal speed in field space, and the slow-roll parameters for $c_I = \frac{1}{2}, 1, 2$ and $n = 12$. Please note that the total transition times are well approximated by equation (3.34), which predicts $Ht_\star \approx 41, 22$, and 12 respectively for each of the parameter values above. Due to the exponential condensation of the flat direction, the dominant effects of the transition will be localized to a much shorter period of time.

As discussed in length in section 3.3.1, we map the longitudinal projection of this two-field system to a single-field system with a potential with a step,

$$V_f(\varphi) = \left(\tilde{V}_0 + \tilde{V}_1\varphi + \tilde{V}_2\varphi^2\right)\left(1 - c_f \tanh\left(\frac{\varphi - \varphi_f}{d_f}\right)\right), \tag{3.57}$$

by determining the parameters $\tilde{V}_0, \tilde{V}_1, \tilde{V}_2, c_f, \varphi_f, d_f$ by numerically fitting $V_f(\varphi)$ to $V(\phi_\parallel \,|\phi_\perp = 0)$, where $\phi_\parallel$ is defined through equation (3.42).

While $\tilde{V}_0, \tilde{V}_1, \tilde{V}_2$ only differ insignificantly from the inflaton potential parameters V_0, V_1, V_2, the transition inscribes a small step in the overall value of the potential, changing its value by at most a few percent. The system (3.32) under the influence of (3.55) can be regarded as a local approximation of the inflaton potential around the location at which the transition occurs, and, as such, it does not describe the full inflationary dynamics until the end of inflation and beyond. In particular, this means that in this simplified model, constraints on the location of the effective step in the inflaton potential (determined by φ_f) cannot be meaningfully interpreted as a constraint on any model parameters. However, constraints on the width (d_f) and the size (c_f) of the step do provide constraints on $c_I^{(0)}$ and n.

In figure 3.2, we plot the the best fit values for c_f and $\frac{c_f}{d_f}$ for $c_I^{(0)} \in \{\frac{1}{2}, 1, 2\}$ and $n \in \{4, \ldots 12\}$, obtained numerically from simulating the system (3.55) in `Mathematica`, and thereafter fitting $V_f(\varphi)$ to $V(\phi_\parallel \,|\phi_\perp = 0)$. The dashed lines in figure 3.2 correspond to the best-fit-values, i.e. $c_f = 1.6{\cdot}10^{-4}$ and $c_f/d_f = 1.7{\cdot}10^{-2}$, together with the one-sigma constrained errors adapted from one-dimensional sections of the likelihood function of the small-field model in [119]. Since the constrained errors are given by sections of the 68% confidence curve, they only provide lower bounds on the errors. In particular, at a larger confidence level, the likelihood function is expected to plateau towards a vanishing step, thus

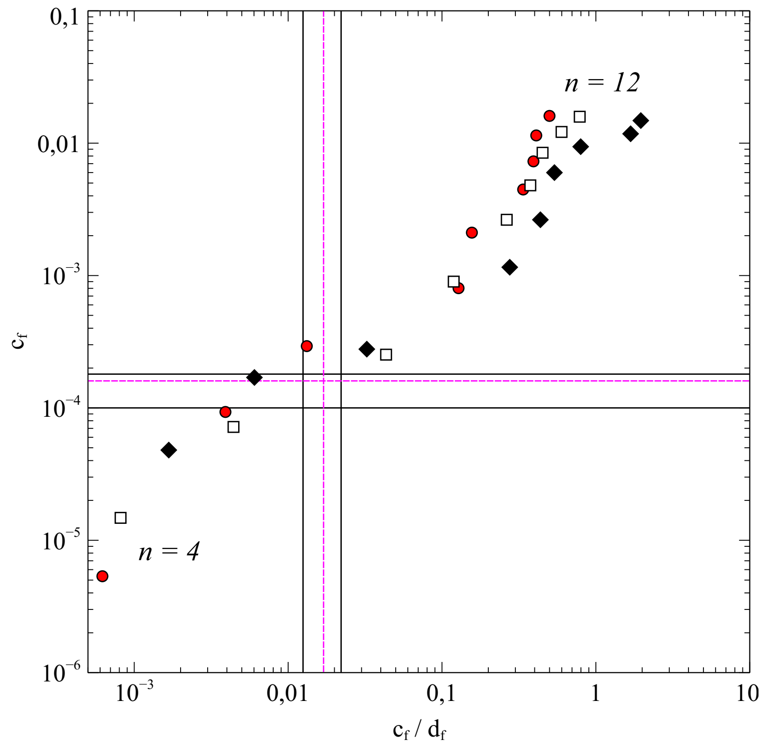


Figure 3.2: The best-fit value of c_f and $\frac{c_f}{d_f}$ (dashed lines) together with the *constrained* errors for the small-field model of [119]. For $n \in \{4, \ldots, 12\}$, the red circles, white squares and black diamonds correspond to the best-fit values of the parameters for $c_I^{(0)} \in \{\frac{1}{2}, 1, 2\}$, respectively. In all cases, $c_I^{(1)} = .1$, $\lambda = 1$, and the Hubble constant just before the transition is $H = 10^{-6} M_{Pl}$, consistent with the COBE normalization of the potential.

only providing an upper bound on the values c_f and $\frac{c_f}{d_f}$, [122]. For $n \lesssim 5$, all points lie in the lower left quadrant, and thus may not be constrained at a higher confidence level. Figure 3.2 illustrates however, that for this subset of Affleck-Dine models of baryogenesis, bounds on features in the potential can impose severe and interesting constraints on the scenario and may prove useful in singling out the flat sector responsible for the generation of baryon number.

Let us discuss the limitations of our analysis in detail: While the small-field inflationary model with a step analyzed in [119] using QUaD, ACBAR and WMAP five-year as well as seven-year data resulted in an improved fit[5] with an effective χ^2_{eff} of about 7 to 9, a similar analysis in the two-field case can be expected to differ in many details. In the context of multiple inflation, this question has been addressed in the works [124, 125], where the inclusion of the transitioning field did not result in an as large improvement of the fit for a model with $H_I \simeq 10^{-8} M_{Pl}$ and for n taking on half-integer values between 7 and 9.5. However, it would be interesting to extend these works to broader classes of inflationary models and to a broad range of transitioning fields.

The largest limitation to this analysis comes from the particular assumptions made about the initial configuration of the system. We will therefore now turn to the question of naturalness of the chosen initial conditions, as well as the feasibility of extracting correlated predictions from the flat direction backreaction on the inflaton for more general initial configurations.

3.3.3 Initial Conditions

The transitioning Affleck-Dine condensate will necessarily backreact on the inflaton potential; however, these effects will be observationally irrelevant if the transition ends before the cosmological scales probed by the CMB left the horizon. For the transition to leave an observable imprint it should terminate between 60 and 50 e-folds before the end of inflation. Clearly, any statement about the naturalness of this happening depends on the physics before observable inflation, out of which very little is known, and what is known is model dependent. In this section we discuss different assumptions about the initial configuration of the system, and provide bounds from above on the duration of the

[5]For a discussion of the significance of the improved fit, see [134]

period in which transitions are likely. For a determination of the likelihood of a flat direction transitioning during the observable period of inflation, this analysis should be supplemented with an embedding of the inflationary scenario in a UV-complete theory, like string theory. This last task is not addressed in this thesis.

Independently of baryogenesis, we find that for large-field inflation, a particular manifestation of the supergravity η-problem restricts the form of the metric on the visible sector field space to be an approximate solution of the equation,

$$R_{m\bar{n}a\bar{b}} = A\ \tilde{K}_{m\bar{n}}\tilde{K}_{a\bar{b}}\,, \tag{3.58}$$

while Affleck-Dine baryogenesis is only viable for certain values of the constant A.

Small-field inflation

Let us first discuss small-field inflation, such that $\Delta\phi \ll M_{Pl}$ during inflation *and* all the way until $H \sim m_{EW}$. We will discuss the case when the Affleck-Dine field starts out in thermal equilibrium with a vanishing vev separately from the case when case when the vev is large enough for the equilibrating interactions to freeze out. The former case is essentially the framework of multiple inflation, discussed in [115].

At the onset of inflation, a flat direction can be trapped in a thermal potential of temperature T_i, which can be no larger than $T_i \sim (H_I M_{Pl})^{1/2}$ in order for the inflaton to dominate the energy density of the universe. In the inflating background the temperature drops exponentially with the number of e-folds, becoming smaller than the Hubble scale of inflation after $N_T \sim \frac{1}{2}\ln\left(\frac{M_{Pl}}{H_I}\right)$ e-folds. For

Hubble scales between $10^{-8}M_{Pl}$ and $10^{-4}M_{Pl}$, this period therefore only lasts for about 5 to 10 e-folds. As the temperature drops below the scale H, the Hubble-induced terms become important, and the value of the holomorphic bisectional curvature between the flat direction and the inflaton— and thereby c_I — determines whether the flat direction transitions or not. For $c_I \lesssim -1$, the flat direction remain trapped at the origin in field space even after the thermal effects have ceased to be important. As we have emphasized, c_I is in general a function of the inflaton vev, but for small-field inflation, the leading order contribution in the ϕ/M_{Pl} expansion should well approximate the true value of c_I throughout the inflaton trajectory.

In case $c_I > 0$, the field will transition as soon as the thermal effects become unimportant. The full transition typically takes several e-folds to complete, and the transition time is logarithmically sensitive to the initial value of the field ψ_i. To estimate the initial vev of thermally trapped field, we note that the displacement from the origin will be a result of criticality in the period when the temperature and curvature contribution to the mass approximately cancel. The two-point correlation function in de Sitter space grows for long wave-length fluctuations [139],

$$\langle\psi^2\rangle \simeq \frac{H^2}{4\pi^2}Ht\,. \tag{3.59}$$

As an estimate, the period of criticality occurs for between $T_i^2 = 2H^2$ and $T_f^2 \simeq \frac{1}{2}H^2$, which corresponds to period of about a ln(2) fraction of an e-fold. During this period, the quantum fluctuations of the condensate results in a vev of the condensate of the order of,

$$\langle\psi^2\rangle^{1/2} \simeq \frac{H}{2\pi}\sqrt{\ln(2)} \approx \frac{1}{10}\,H\,. \tag{3.60}$$

The analytic solution of ψ for small vevs, equation (3.34) then gives an approximation and a lower bound for the transition time,

$$\Delta N \approx \frac{2(n-3)}{3(n-2)} \frac{1}{\sqrt{1+\frac{8}{9}c_I}-1} \ln\left(\frac{M_{Pl}}{\psi_i}\right), \qquad (3.61)$$

which, as an example, for $\psi_i = H = 10^{-6} M_{Pl}$, $n = 12$, and $c_I = (\frac{1}{2},\ 1,\ 2)$ evaluates to $\Delta N \approx (12,\ 21,\ 39)$, in good agreement with numerical simulations, c.f. Table 1. Furthermore, for $c_I \leq \frac{1}{3}$, the transition period lasts for at least 50 e-folds for $n \geq 6$, and for more than 60 e-folds for $n \geq 12$, which demonstrates that transitioning flat directions can also be relevant far past the beginning of the inflationary era. Since the condensation develops exponentially in time, most of the field excursion of the flat direction, and thereby most of the effects of the transition, will be confined to a much shorter period of time of a few e-folds before and after the end of the transition. As a demonstration of this fact, we note that by the numerical analysis of the preceding section (as is illustrated by the red circles in figure 3.2), even transitions taking as long as 40 e-folds in total, give rise to localized features in the effective inflaton potential $V_f(\varphi)$, which are large enough to be easily excluded by CMB data.

For a model like the MSSM with many flat directions, there can be a number of flat directions eligible for transition by all having $c_I > 0$. The field with the maximal c_I will transition first, and in the process, lift all flat sectors it does not belong to through the renormalizable superpotential, leaving a smaller subset of flat directions still eligible for transition. For a mass estimate of the lifted directions, see §3.4.3. This suggests that c_I may in fact have a typical value that is larger than unity for the flat direction that transitions first. For $O(c_I) \simeq 10$, the transition is prompt and over within the first five e-folds after the temperature drops below H. Other flat directions in the same sector may also transition during the same period, and thus may extend the period during which it is reasonable to expect transitioning flat directions.

It is interesting to ask whether the transition of a flat direction may "trigger" the transitions of other flat directions, which presumably could extend the total transition period. In §3.4.3, we discuss the different interactions in a flat sector, and classify under what conditions such a triggering may occur. In sum, in the absence of non-holomorphic tri-linears that can induce negative contributions at either quadratic and linear order in the fields, triggering is potentially possible through superpotential couplings between different flat directions. As we discuss in §3.4.3, triggering is only possible if these superpotential operators satisfy certain conditions.

In conclusion, we find that for small-field inflation with initially thermally trapped flat directions, a mild fine-tuning of c_I of an order of magnitude is sufficient to extend the transition time to more than 50 e-folds for most flat directions, and for $c_I = 1$, transitions can be expected during the first 12 to 22 e-folds after the thermal potential has subdued — or equivalently, for the first 20 to 30 e-folds of inflation — for $4 \leq n \leq 12$. Since c_I is essentially constant in slow-roll inflation, the number of transitions is bounded from above by the number of fields in the flat sector that transitions.

Turning to the large class of small-field inflationary model in which the flat directions are *not* thermally trapped at the onset of inflation, we first consider the case where one or several of the flat directions initially have large vacuum expectation values with $\psi_i > \psi_f$. The relaxation of these fields will again result in a 'backreaction' on the inflaton potential. Focusing on the period of slow-roll inflation during which the inflaton ϕ is assumed to dominate the energy density of the universe, the initial vevs of the fields are bounded from above by $\psi_{\max} \sim M_{Pl}\left(\frac{H}{M_{Pl}}\right)^{\frac{1}{n-1}}$, as discussed in [97]. The potential is steeper for fields transitioning from large vevs to ψ_f, than for those transitioning from close to the origin in field space, and consequently the period during which it is reasonable to expect relaxation of flat directions is more limited. Also in this case, multi-field effects such as "triggering" can be very important and may prolong the period during which it is reasonable to expect condensates to develop, but the details are highly model-dependent.

In general, an entire flat sector will transition at the onset of inflation: some to the origin and some to the minimum ψ_f (the magnitude of which may of course differ between the different flat directions, since they can be lifted at different orders and are subject to different coupling constants), depending on the sign of the respective c_I's. If all $c_I \geq 1$, and in the absence of multi-field effects, these transitions will be over within the 20 first e-folds, from which it follows that the backreaction on the inflaton during this period can be substantial. Since multiple fields may transition simultaneously, the induction of "bumps" in the inflaton potential is not at all unlikely at the early stages of inflation [125].

Large-field inflation

Let us now discuss the naturalness of observing transitions during large-field inflation, by which we mean inflationary scenarios in which the inflaton field excursion from the onset of inflation and until $H \approx m_{EW}$ is of order M_{Pl} or larger. Such models suffer more severely from the supergravity η-problem, and the Affleck-Dine mechanism analogously requires a very special form of the Kähler potential, as we will now discuss.

An initially thermally trapped field will experience a thermal potential during the first couple of e-folds of inflation after which the exponentially decreasing temperature drops below H_I, and the curvature induced masses become important. After this point the fields may transition — just as in small-field inflation — if the corresponding holomorphic bisectional curvature so permits. However, crucially, the holomorphic bisectional curvature (and thereby c_I), typically experience an $O(1)$ variation over a field excursion of M_{Pl}. This is exemplified by the simplest possible model of the Affleck-Dine mechanism in supergravity, cf. equation (3.16), which successfully can be embedded in small-field inflation, but not in large-field inflation.

For monomial potentials of the form $m^2\phi^2$, the field excursion from the last 60 e-folds and until the inflaton settles down after reheating is a distance of 15 M_{Pl} in field space. Thus, during this period it is perfectly natural to expect the value of c_I to change, possibly several times, and for the flat direction to make a number of excursions during inflation. In fact, since in this case transitions every 4 or so e-folds may occur, the Affleck-Dine fields will cause significant deviations from a scale invariant primordial spectrum of fluctuations on all scales. Since this is not observed, clearly c_I cannot be a generic function of the inflaton vev and the Kähler potential must be of some restricted form.

Similarly, if the flat directions are not thermally trapped at the onset of inflation, transitions can be expected at any point during inflation.

We conclude that in large-field models the transitions of the Affleck-Dine field are generically ubiquitous, however the non-observance of significant deviations from scale invariance in the primordial power spectrum is suggestive. If the physics of the early universe was indeed governed by F-term large-field inflation, then $B[\phi,\psi]$ should be at least approximately constant, *with or without Affleck-Dine baryogenesis*. This in turn requires — at least to a good approximation — the Riemann tensor to take the form[6],

$$R_{m\bar{n}a\bar{b}} = A\ \tilde{K}_{a\bar{b}}\tilde{K}_{m\bar{n}}\,, \tag{3.62}$$

evaluated at $\psi^a = 0$, as always. The scalar A is in general an inflaton dependent function, but for $B[\phi,\psi]$ to be slowly varying over Planckian distances, A should similarly be slowly varying and can, at least approximately, be treated as a constant. In the simplified case of a single visible sector field, ψ, and a single inflaton, ϕ, equation (3.62) is the Poisson equation in the flat ϕ-plane for $f(\phi,\bar{\phi}) = \ln \tilde{K}_{\psi\bar{\psi}}$,

$$\nabla^2_{(\phi)} f(\phi,\bar{\phi}) = A\tilde{K}_{\phi\bar{\phi}}(\phi,\bar{\phi})\,. \tag{3.63}$$

Denoting the Green's function in the domain D of the ϕ-plane by $G(\phi,\phi')$, we find that

$$f(\phi,\bar{\phi}) = A\tilde{K}(\phi,\bar{\phi}) - A\oint_{\partial D} ds'\,\tilde{K}(\phi',\bar{\phi}')\partial_n G(\phi,\phi')\,, \tag{3.64}$$

where we have assumed that the Green's function vanish on the boundary of D. If $\tilde{K}$ can be made to vanish on ∂D by a suitable Kähler transformation, the condition of approximately constant masses further simplifies to,

$$\tilde{K}_{\psi\bar{\psi}}(\phi,\bar{\phi}) = e^{A\tilde{K}(\phi,\bar{\phi})}\,. \tag{3.65}$$

Similar equations can be derived for multiple visible sector fields, if the metric $\tilde{K}_{a\bar{b}}$ has some special structure.

Motivated by the constant c_I found in the string theory realizations of §3.2.2, we can without any loss of generality write the full Kähler potential of the form

[6]Note however that $R_{a\bar{b}c\bar{d}}$, and $R_{m\bar{n}l\bar{k}}$ are not necessarily proportional to the corresponding metric-elements.

$K = -\alpha \ln U$, where U is a real function of the visible and hidden sector fields and α is a constant. In terms of this parametrization of the Kähler potential, the relevant components of the Riemann tensor are

$$R_{m\bar{n}a\bar{b}} = \frac{1}{\alpha}\tilde{K}_{a\bar{b}}\tilde{K}_{m\bar{n}} + V_{a\bar{b}m\bar{n}}\,, \tag{3.66}$$

where

$$V_{a\bar{b}m\bar{n}} = -\frac{\alpha}{U}\left(U_{a\bar{b}m\bar{n}} - \left(U^{-1}\right)^{\bar{c}d} U_{a\bar{c}m}U_{\bar{b}d\bar{n}}\right). \tag{3.67}$$

Clearly, the tensor $V_{a\bar{b}m\bar{n}}$ has to be either proportional to the product of the metrics on the moduli space, or vanish in order for the condition (3.62) to hold. For example, in no-scale supergravity and Kähler potentials of the sequestered form, $\alpha = 3$ and $V_{a\bar{b}m\bar{n}}$ vanish since the function U is separable, c.f. equation (3.25). We discussed in §3.2.2 how these types of models indeed gives rise to a constant holomorphic bisectional curvature, but do not allow for Affleck-Dine baryogenesis. Indeed, for $V_{a\bar{b}m\bar{n}}$ vanishing, successful Affleck-Dine baryogenesis bounds the constant α from above,

$$\alpha < \begin{cases} \frac{3}{2} & \text{if } m_{3/2}^2 \ll H^2 \\ 1 & \text{if } m_{3/2}^2 = H^2\,. \end{cases} \tag{3.68}$$

For a single visible sector field and a single inflaton, equation (3.65) imply that $U_{\psi\bar{\psi}}$ is a constant and that the tensor $V_{\psi\bar{\psi}\phi\bar{\phi}}$ vanish if $\tilde{K}$ can be chosen to vanish on ∂D.

3.4 Further Correlated Predictions

The holomorphic bisectional curvature appears in a number of different places in the supergravity Lagrangian, and the geometric condition of the Affleck-Dine scenario discussed in §3.2.1 thus leads to a number of definite correlated predictions for various couplings. In this section we discuss (classical) wave-function renormalization, fermion terms, and couplings between multiple flat directions.

3.4.1 Wave-function normalization

The function c_I of equation (3.13) determines the canonically normalized mass of a flat direction, and thereby accounts for any changes in the metric on the moduli space $\tilde{K}_{\psi\bar{\psi}}$. In the simplified model we used for numerical analysis in §3.3.2 however, we assumed that the inflaton and flat direction were canonically normalized throughout the relevant part of inflation, despite the fact that the kinetic terms can be expanded as,

$$\left(1 - B[\phi,\psi]\left(\frac{\psi}{M_{Pl}}\right)^2 - \frac{1}{4}B[\phi,\phi]\left(\frac{\Delta\phi}{M_{Pl}}\right)^2\right)\partial_\mu\phi\partial^\mu\phi^*\,, \tag{3.69}$$

$$\left(1 - B[\phi,\psi]\left(\frac{\Delta\phi}{M_{Pl}}\right)^2 - \frac{1}{4}B[\psi,\psi]\left(\frac{\psi}{M_{Pl}}\right)^2\right)\partial_\mu\psi\partial^\mu\psi^*\,, \tag{3.70}$$

where the bisectional curvatures are evaluated at some reference point ϕ_0 along the inflationary trajectory at which the fields are canonically normalized, and $\Delta\phi = \phi - \phi_0$. This omission is well motivated: for small field inflation $\Delta\phi \ll M_{Pl}^2$, and the inflaton-dependent correction can be neglected, and while the transitioning flat direction will induce a change in normalization for both the flat direction itself and for the inflaton, this results in small changes in the parameters of the inflaton potential and negligible changes in c_I and λ for the flat direction. For concreteness, in the model of §3.3.2 with potential given by equations (3.55) and (3.56), the change in canonical normalization of the inflaton due to the transitioning field leads to a redefinition of the parameters of the inflaton potential of the form

$$
\begin{aligned}
V_0 &\rightarrow V_0\,, && (3.71)\\
V_1 &\rightarrow \left(1 - B[\phi,\psi]\left(\frac{H}{M_{Pl}}\right)^{\frac{2}{n-2}}\right)^{-\frac{1}{2}} V_1\,, && (3.72)\\
V_2 &\rightarrow \left(1 - B[\phi,\psi]\left(\frac{H}{M_{Pl}}\right)^{\frac{2}{n-2}}\right)^{-1} V_2\,. && (3.73)
\end{aligned}
$$

As a result, the inflaton potential for the canonically normalized field will appear slightly tilted after the transition of the flat direction. Nevertheless, the effect of the field-redefinition is small for most values of n, which motivates the omission of this effect in §3.3.2.

3.4.2 Fermion couplings

The Riemann tensor on the field space is well-known to appear in the quartic fermion quartic couplings. The holomorphic bisectional curvature will thus necessarily gives rise to an additional coupling between the visible sector fermions (denoted χ^a), and fermions in the multiplet of some hidden sector field (denoted χ^m),

$$
\begin{aligned}
\frac{1}{\sqrt{-g}}\mathcal{L} &= -\frac{1}{4}\left(\frac{1}{M_{Pl}^2}K_{m\bar{n}}K_{a\bar{b}} + 2R_{a\bar{b}m\bar{n}}\right)\chi^m\chi^{\dagger\bar{n}}\chi^a\chi^{\dagger\bar{b}}\\
&= -\frac{1}{4M_{Pl}^2}\left[1 - 2B(\phi,\psi)\right]\tilde{K}_{\phi\bar{\phi}}\tilde{K}_{\psi\bar{\psi}}\chi^\phi\chi^{\dagger\bar{\phi}}\chi^\psi\chi^{\dagger\bar{\psi}}\,,
\end{aligned}
\tag{3.74}
$$

for non-canonically normalized fields. In the last step of equation (3.74), we have again specialized to a single flat direction and a single inflaton. However, even though the holomorphic bisectional curvature predicted by the Affleck-

Dine mechanism changes the numerical value of this coupling, it is hardly important for the physics of the early universe.

For completeness as well as for the potential embeddings of fermionic preheating in a complete supergravity model, the masses of the flat direction fermions receive contributions from *both* supersymmetric and supersymmetry breaking sources during inflation, which for the non-canonically normalized field is given by,

$$m_{\psi^a} \quad = \quad e^{K/2M_{Pl}^2}\mathcal{D}_\psi D_\psi W \,. \tag{3.75}$$

At vanishing vev of ψ, its fermionic partner is massless, while for a displaced flat direction with vacuum expectation value ψ_f, the mass is to leading order in $\frac{\psi_f}{M_{Pl}}$ given by,

$$\begin{aligned} m_\psi \quad &= \quad \tilde{K}_{\psi\bar{\psi}}\left[\left(\frac{W}{|W|}m_{3/2} - k_1\sqrt{H^2 + m_{3/2}^2}\right)\tilde{K}_{\psi\bar{\psi}}(\frac{\psi_f^*}{M_{Pl}})^2 \right. \\ &+ \quad \left. k_2\sqrt{H^2 + m_{3/2}^2}\tilde{K}_{\psi\bar{\psi}}|\frac{\psi_f}{M_{Pl}}|^2\right] , \end{aligned} \tag{3.76}$$

where

$$k_1 \;=\; \sqrt{3}M_{Pl}^3\frac{F_\phi}{|F_\phi|}\left(\tilde{K}^{\phi\bar{\phi}}\right)^{1/2}\left(\tilde{K}^{\psi\bar{\psi}}\right)^2\partial_\phi\tilde{K}^{(2,2)}_{\psi\bar{\psi}\psi\bar{\psi}} \,, \tag{3.77}$$

and,

$$k_2 \;=\; \sqrt{3}M_{Pl}^3\frac{F_\phi}{|F_\phi|}\left(\tilde{K}^{\phi\bar{\phi}}\right)^{1/2}\left(\tilde{K}^{\psi\bar{\psi}}\right)^3\tilde{K}^{(2,2)}_{\psi\bar{\psi}\psi\bar{\psi}}\partial_\phi\tilde{K}_{\psi\bar{\psi}} \,. \tag{3.78}$$

The mass of the canonically normalized fermion is therefore of the order of $H\left(\frac{H}{M_{Pl}}\right)^{\frac{2}{n-2}}$ during inflation.

After inflation, once $H \simeq m_{3/2} \simeq m_{EW}$, the mass of the flat direction fermion is of the order of $m_{3/2}\left(\frac{m_{3/2}}{M_{Pl}}\right)^{\frac{2}{n-2}} \ll m_{3/2}$.

3.4.3 Consequences for multi-field dynamics

Any supersymmetric extension of the Standard Model will involve many renormalizably flat directions — in the MSSM they add up to hundreds — and the Affleck-Dine mechanism is consequently concerned with a multi-field system. In this section we survey the leading couplings between different flat directions with a particular focus on the interactions in multi-dimensional flat sectors, as well as the possibility of a developing Affleck-Dine condensate "triggering" a transition of other flat directions. We discuss how triggering may proceed through superpotential interactions or through the supergravity induced non-holomorphic tri-linear C-term.

To avoid unnecessary cluttering, we will in this section assume that $e^{\tilde{K}/M_{Pl}^2} \simeq 1$ and that all fields are canonically normalized. It is not hard to generalize the following equations to the more general case.

Interactions induced by the renormalizable superpotential

While any flat direction per definition is F- and D-flat in global, Minkowski, supersymmetry with respect to the renormalizable superpotential, this does not mean that all flat directions can simultaneously obtain large vacuum expectation values. The renormalizable superpotential introduces interactions between different flat directions, and the presence of a condensate can significantly restrict the number of dynamically interesting fields.

For example, in the MSSM both the $\Psi_1^2 = H_u H_d$ operator and the $\Psi_2^3 = L_1 L_3 e_1$ operator correspond to supersymmetric flat directions. The lepton Yukawa coupling,

$$W \supset \lambda_e^{ij} H_d L_i e_j \,, \tag{3.79}$$

gives rise to an F-term for L_i^α, (here α is an $SU(2)$ index and i a flavor index), which in global supersymmetry is given by,

$$F_{L_i^\alpha} = H_d^\alpha e_i = 0 \,, \tag{3.80}$$

where invertibility of the lepton Yukawa matrix has been assumed. Clearly, simultaneous vacuum expectation values of Ψ_1 and Ψ_2 give rise to a non-vanishing F-term.

In general, the renormalizable superpotential contains couplings between different flat directions of the form,

$$W \supset \lambda \Psi_1 \Psi_2^2 \,, \tag{3.81}$$

which, in the globally supersymmetric scalar potential, results in a quartic coupling of the form

$$V(\psi_1, \psi_2) \supset 4|\lambda|^2 |\psi_1|^2 |\psi_2|^2 \,. \tag{3.82}$$

If either of the fields transitions to a magnitude of ψ_f, as in equation (3.4), the other will obtain a mass-squared of the order of

$$|\lambda|^2 M_{Pl}^2 \left(\frac{H}{M_{Pl}}\right)^{\frac{2}{n-2}} = |\lambda|^2 H^2 \left(\frac{M_{Pl}}{H}\right)^{\frac{n-4}{n-2}} \,, \tag{3.83}$$

where λ denotes to the relevant Yukawa coupling. For small n, and for the smaller of the Yukawa couplings, this contribution of the mass is much smaller than the Hubble constant during inflation, and thus a subleading contribution to the total mass of the flat direction, while for larger n and for Yukawa couplings $\lambda \gtrsim 10^{-3}$, it can lead to induced masses of the order of Hubble or larger.

In the former case, we would regard the fields to belong to the same flat sector with respect to the renormalizable superpotential, while in the latter case they would belong to different sectors. To our knowledge, there is no complete classification of simultaneous flat directions, however see [106, 105] for some interesting special cases.

Order-n A-terms

The most important contribution from the non-renormalizable superpotential terms of the form (3.9) is to generate the stabilizing term proportional to $|\psi|^{2n-2}$ in the scalar potential, as well as the order-n A-term, as in (3.3). In supergravity, the A-term arises from contributions to the scalar potential of the form

$$\begin{aligned} V_F &\supset \frac{1}{n}\Big(\tilde{\bar{F}}_{\bar\phi}\tilde{K}^{\phi\bar\phi}(D_\phi\lambda)M_{Pl}^3 - 3\lambda\frac{\tilde{W}^*}{|\tilde{W}|}m_{3/2}M_{Pl}^3\Big)\left(\frac{\psi}{M_{Pl}}\right)^n \\ &= \frac{1}{n}\Big(\sqrt{3(H^2+m_{3/2}^2)}\frac{\tilde{\bar{F}}_\phi}{|\tilde{\bar{F}}_\phi|}D_\phi\lambda - 3\lambda\frac{\tilde{W}^*}{|\tilde{W}|}m_{3/2}M_{Pl}^3\Big)\left(\frac{\psi}{M_{Pl}}\right)^n . \end{aligned} \tag{3.84}$$

where we have specialized to a single hidden sector field and neglected higher order terms in Kähler potential of type $\tilde{K}^{(n,0)}$ contribute with $\frac{H}{M_{Pl}}$ suppressed corrections to the coupling (3.84).

Triggering

More interesting are the contributions to the non-renormalizable superpotential that mix different flat directions, as in equation (3.9). Cross-couplings between flat directions in the same sector can be present at order $k > 3$. For concreteness, suppose that the field ψ_1 appears at quadratic order in a contribution to the superpotential at order k which couples it to a second flat direction ψ_2, i.e.

$$W \supset \frac{\lambda^{(k)}}{k} M_{Pl}^{3-k} \Psi_1^2 \Psi_2^{k-2} \,. \tag{3.85}$$

If ψ_2 obtains a vev of the order of ψ_f, then the resulting mass-squared for the field ψ_1 will obtain a positive definite contribution of the form,

$$|\lambda^{(k)}|^2 H^2 \left(\frac{2}{k}\right)^2 \left(\frac{H}{M_{Pl}}\right)^{\frac{2(k-n)}{n-2}} \,. \tag{3.86}$$

The superpotential coupling (3.85) also gives rise to an order-k holomorphic A-term in the scalar potential of the type (3.84). After ψ_2 condenses, this coupling will schematically contribute to the ψ_1 mass with,

$$\frac{|\lambda^{(k)}|}{k} H \sqrt{H^2 + m_{3/2}^2} \left(\frac{H}{M_{Pl}}\right)^{\frac{k-n}{n-2}} \cos(\beta) \,, \tag{3.87}$$

The angle β depends on the phases of ψ_1, ψ_2 and $D_\phi \lambda^{(k)}$, and — in the absence of other contributions to the mass of ψ_1 — the cosine dependence of (3.87) will cause two tachyonic directions to open up in the ψ_1 plane, under the condition that

$$\frac{1}{k} |\lambda^{(k)}| \left(\frac{H}{M_{Pl}}\right)^{\frac{k-n}{n-2}} \gtrapprox 1 \,. \tag{3.88}$$

For $k < n$, this equation requires fine-tuning of the coefficient $\lambda^{(k)}$ to be satisfied, and triggering due to condensation does not appear to be generic. For $k > n$ on the other hand, equation (3.88) only imposes mild restrictions on the value of $\lambda^{(k)}$ to be satisfied. In this case however, the magnitude of the tachyonic contribution to the mass in equation (3.87) is suppressed with respect to H^2, and may not render the total mass-squared negative.

"C-terms"

The phenomenological potential (3.3) gives a clear view of the dominant effect for Affleck-Dine baryogenesis involving a single field. For multiple fields however, there are more terms that potentially can produce interesting effects. In particular, the nontrivial Kähler potential can induce terms at all orders, and thus introduce operators with a much lower dimension than those originating from the superpotential. For flat directions, the lowest dimensional contributions of this sort arise at cubic order, in terms of the non-holomorphic tri-linears sometimes called "C-terms" [10],

$$V_F \supset \frac{1}{2} c^{(2,1)}_{ab\bar{c}} \, \psi^a \psi^b \psi^{*\bar{c}} + c.c.\,. \tag{3.89}$$

As we will see, C typically is of the order $H\frac{H}{M_{Pl}}$, and the C-term gives rise to a mass term of the order of

$$H^2 \left(\frac{H}{M_{Pl}} \right)^{\frac{1}{n-2}} , \tag{3.90}$$

for a flat direction appearing quadratically in the C-term, coupling to another flat direction that is lifted at order n with vev given by (3.4). On the other hand, a field appearing linearly in the C-term may experience a *linear* instability of the order of

$$H^2 M_{Pl} \left(\frac{H}{M_{Pl}} \right)^{\frac{2}{n-2}} , \tag{3.91}$$

if the other field appearing in the C-term have condensed.

This type of non-holomorphic — sometimes called "maybe soft" — tri-linears are severely restricted by gauge invariance and R-parity [10]. For instance, in terms of the ordinary MSSM fields, the only operators of this form are [140]

$$\tilde{Q}\tilde{u}H_d^* ,\; \tilde{Q}\tilde{d}H_u^* ,\; \tilde{L}\tilde{e}H_u^* . \tag{3.92}$$

To assess the importance of these operators, we first note that all the C-terms involve a Higgs field and thus couples the only two flat directions including the Higgs fields, $H_u H_d$ and $H_u L_i$, to operators involving squarks or sleptons. For the $H_u H_d$ direction, F-flatness of the renormalizable superpotential requires both $\tilde{Q}_i = 0$ and $\tilde{e}_i = 0$, thus enforcing the vanishing of all C-terms coupling to the $H_u H_d$-direction. In other words, for the $H_u H_d$ operator, the C-terms lift no directions that are not also lifted by the renormalizable superpotential.

The $H_u L_i$ direction is simultaneously F-flat with some operators of the form $L_i L_j e_k$, and studying this sector in detail — including the possibility of a non-vanishing C-term — could be very interesting.

The supergravity induced C-terms can easily be given a geometric interpretation, since when phrased in terms of the covariant fluctuation obtained by the background field method, explained in [141], they are given by

$$\begin{aligned} \hat{c}^{(2,1)}_{ab\bar{c}} &= \frac{2}{3}\left(\nabla_a \nabla_b \nabla_{\bar{c}} V_F + \nabla_a \nabla_{\bar{c}} \nabla_b V_F + \nabla_{\bar{c}} \nabla_a \nabla_b V_F\right) = \\ &= 2\left(\nabla^3_{ab\bar{c}} - \frac{1}{3} R^{B}{}_{a\bar{c}b} \nabla_B\right) V_F \\ &= -2e^{\tilde{K}}\left(F^{\bar{m}} \bar{F}^{n} \nabla_a R_{\bar{m}n\bar{c}b} + \frac{1}{3} R_{a\bar{c}b\bar{m}} (\tilde{K}^{m\bar{m}} \bar{F}^{l} \mathcal{D}_m F_l + F^{\bar{m}} \bar{W})\right), \end{aligned} \tag{3.93}$$

where $\mathcal{D}_m F_l = \partial_m F_l + K_m F_l - \Gamma^{p}_{ml} F_p$, all in natural units.

Since, by gauge and R-parity invariance, there is no allowed cubic self-interaction from the C-terms in the MSSM for any flat direction, we have

$$\hat{c}^{(2,1)}_{\psi\psi\bar{\psi}} = 0\,. \tag{3.94}$$

The three terms contributing to $\hat{c}^{(2,1)}_{\psi\psi\bar{\psi}}$ in (3.93) are in general independent functions of the inflaton which (depending on the inflationary scenario) may even be of different orders of magnitude, and we will not consider the case when there are nontrivial cancellations between them. It then follows that each term has to cancel separately so that at vanishing vev of the flat direction, $\psi = 0$,

$$R_{\psi\bar{\psi}\psi\bar{\phi}} = 0\,, \tag{3.95}$$

and the relevant holomorphic bisectional curvature is covariantly constant along the flat direction,

$$\nabla_{\psi} B[\phi, \psi] = 0\,. \tag{3.96}$$

Quartic interactions

Finally, the quartic interactions couple all flat directions to each other and generically gives rise to contributions to the squared masses of a flat direction of the order of

$$H^2 \left(\frac{H}{M_{Pl}}\right)^{\frac{2}{n-2}}\,, \tag{3.97}$$

in the background of another condensed flat direction, stabilized at order n in the superpotential. In case a given flat direction is not stabilized by any non-renormalizable operator in the superpotential, the quartic self-interactions will stabilize the condensate to a vev of the order of M_{Pl}.

The quartic scalar couplings for the covariant fluctuations, $\hat{\psi}^a$,

$$V_F \supset \frac{1}{4}\lambda_{a\bar{b}c\bar{d}}\hat{\psi}^a\hat{\psi}^{\bar{b}}\hat{\psi}^c\hat{\psi}^{\bar{d}}\,, \tag{3.98}$$

are given by the symmetrized covariant derivative $\frac{1}{3!}\nabla^4_{(a\bar{b}c\bar{d})}V_F$, which can be written in natural units as,

$$\begin{aligned} \lambda_{a\bar{b}c\bar{d}} \;&=\; e^{\tilde{K}}\Big[\left(F_m\bar{F}^m - |W|^2\right)\tilde{K}_{\bar{b}\{a}\tilde{K}_{c\}\bar{d}} + \bar{F}^m F^{\bar{n}}(K^{A\bar{A}}R_{m\bar{n}c\bar{A}}R_{A\bar{b}a\bar{d}} - K^{A\bar{A}}R_{m\bar{n}A\bar{d}}R_{\bar{A}a\bar{b}c} \\ &+\; K^{A\bar{A}}R_{m\bar{A}\{a\bar{d}}R_{A\bar{n}c\}\bar{b}} - K_{\{a\bar{b}}R_{m\bar{n}c\}\bar{d}} - K_{\{a\bar{d}}R_{m\bar{n}c\}\bar{b}} - \nabla^2_{\bar{b}a}R_{m\bar{n}c\bar{d}}) - R_{a\bar{b}c\bar{d}}|W|^2 \\ &-\; W\bar{F}^n\nabla_a R_{n\bar{d}c\bar{b}} - \bar{W}F^{\bar{n}}\nabla_{\bar{b}}R_{\bar{n}a\bar{d}c}\Big] \end{aligned} \tag{3.99}$$

where braced indices are symmetrized (i.e, $T_{\{ab\}\dots} = T_{ab\dots} + T_{ba\dots}$), and capital letters run over both the visible and hidden sectors. For a single inflaton and a canonically normalized flat direction, the expression simplifies to

$$\begin{aligned}\lambda_{\psi\bar{\psi}\psi\bar{\psi}} &= 6H^2\left((1+B[\phi,\psi])^2 + \frac{1}{2}\nabla^2_{|\psi}B[\phi,\psi]\right) - 2m_{3/2}\mathrm{Re}\left(\frac{W}{|W|}\bar{F}^{\phi}\nabla_{\psi}R_{\phi\bar{\psi}\psi\bar{\psi}}\right)\\ &+ m_{3/2}^2\Big(6\,(1+B[\phi,\psi])^2 - 2 + 3\nabla^2_{|\psi}B[\phi,\psi] + B[\psi,\psi]\Big),\end{aligned} \tag{3.100}$$

after using (3.95) and (3.96). Here $\nabla^2_{|\psi}B[\phi,\psi] = \tilde{K}^{\psi\bar{\psi}}\nabla^2_{\psi\bar{\psi}}B[\phi,\psi]$. Evidently, the quartic interaction term depends on the holomorphic bisectional curvature, and thus provide a nontrivial correlation of the scenario.

3.5 Conclusions

The generation of the observed baryon asymmetry is one of the outstanding problems of twentieth century physics which remains unsolved a decade into the twenty-first century. Fortunately, as cosmological observations continue to become ever more exact, one may ask to what extent the rise of precision cosmology can help solve the question of baryogenesis.

In this Chapter, we have explored a prediction of a sub-class of Affleck-Dine scenarios, in which a flat direction transitions from a small vev to a larger vev during the period of inflation when cosmological scales left the horizon. In this case, we found that the near scale invariance of the cosmic microwave background places severe restrictions on the nature of the flat direction.

A two-field system with a transitioning flat direction can be modeled — at least as a rough, first approximation — as a single-field model with a step in the potential. Since the temperature spectrum of the CMB appears to favor a largely featureless inflaton potential, observations of the temperature anisotropies impose strong constraints on the location, the size and the width of any step appearing in the inflaton potential. Interestingly, the ΛCDM cosmological model with a step located at a specific position in the inflaton potential can improve the fit to WMAP data by a marginally significant amount.

In the sub-class of models considered in §3.3, the corresponding constraints on the size and width of the step can be interpreted as constraints on the parameters of the transitioning flat direction, namely the dimension at which the flat direction is lifted in the superpotential, n, and the holomorphic bisectional curvature between the inflaton and the flat direction, denoted $B[\phi, \psi]$, which determines the (tachyonic) mass of the flat direction at the origin in field space.

In the toy model considered in §3.3.2 with a Hubble parameter during inflation given by $H \simeq 10^{-6}\ M_{Pl}$, *no* flat direction with a tachyonic mass in the range $-\frac{H^2}{2}$ and $-2H^2$ and n in the range from 4 to 12 could produce a step in the inflaton potential of the size and width included within the 68% confidence region of the best-fit value. This suggests that transitioning flat directions during inflation can be severely constrained by CMB data. However, our analysis in §3.3.2 should be regarded as a first step towards a better understanding of the cosmological predictions of this sub-class of Affleck-Dine models, and a full analysis, like the one of [124, 125] done in the context of multiple inflation would be very interesting to pursue for a broader range of inflationary models.

Future precision cosmology observations of the E-mode spectrum and, possibly, of a non-vanishing non-Gaussianity of the temperature anisotropies will determine the nature and significance of the features in the temperature spectrum responsible for the improved fit for a potential with a step. Either outcome will provide important information about the naturalness of the Affleck-Dine scenario. Since observations of non-Gaussianities can potentially serve to discriminate between a single-field model with a step and a multiple-field model with a transitioning flat direction (see e.g. [135, 133]), it would certainly be interesting to study the non-Gaussianities produced by a transitioning field in a variety of inflationary models.

From the point of view of string phenomenology, the Affleck-Dine mechanism is a particularly attractive scenario for the generation of the baryon asymmetry. By being sensitive to Planck-suppressed operators whose structure are dictated by the string theory realization, the mechanism can potentially provide clean information about the coupling between the visible sector and the inflationary sector. For instance, if the Affleck-Dine mechanism indeed is responsible for the observed baryon asymmetry, then it immediately follows that the early universe cannot be described by brane inflation together with a sequestered visible sector, as we have discussed in §3.2.2.

In this Chapter, we have also elaborated on the possibility of extracting correlated prediction from the nontrivial structure of $\mathcal{N} = 1$ supergravity. In particular we have discussed fermion couplings, multi-field couplings and higher-order Planck-suppressed interaction terms, which are completely or partially correlated with the magnitude of the tachyonic mass of the flat direction at the origin in field space through the holomorphic bisectional curvature that appear repeatedly in the supergravity Lagrangian.

An attractive feature of the Affleck-Dine mechanism is that it not only solves the problem of baryogenesis, but that it appears to provide a robust frame-work for production of dark matter and for explaining the approximate coincidence of the dark matter and the baryon densities. If supersymmetry is relevant for the description of our universe, then the Affleck-Dine mechanism could very well play a key role in the unification and solution of several cosmological problems.

CHAPTER 4

THE WASTELAND OF RANDOM SUPERGRAVITIES

4.1 Introduction

Perhaps the most pressing question in string theory is whether the theory admits solutions consistent with all observations. In light of the discovery of the acceleration of the universe, it is essential to pursue de Sitter solutions of string theory, and to understand whether these solutions are so numerous that they can account for the smallness of the vacuum energy. After a decade marked by significant advances in understanding flux compactifications [142], there is now compelling, but still largely indirect, evidence for the existence of a vast landscape of metastable de Sitter vacua. Direct enumeration of explicit de Sitter vacua remains a distant goal.

The cardinal difficulty in constructing de Sitter solutions is that in the absence of supersymmetry, the scalar potential can have instabilities along one or more directions in the scalar field space. When the number of fields is large – which is both generic in Calabi-Yau flux compactifications, and indispensable for providing an astronomical number of vacua – direct examination of the Hessian matrix of the scalar potential becomes impractical. This impasse motivates a statistical approach, in which the compactification data are taken to be random variables.

As a metastable vacuum is a critical point of the scalar potential at which the Hessian matrix is positive definite, it is natural first to ascertain the statistical properties of general critical points, and then to characterize the subset of critical points that are in fact metastable vacua. In the seminal work [33], Denef and Douglas formulated this problem for a general four-dimensional $\mathcal{N} = 1$ supergravity theory, taking the superpotential W and Kähler potential K to be random functions, in a precise sense that we shall review. Denef and Douglas studied the eigenvalues of the Hessian matrix $\mathcal{H}$ and argued that a significant fraction of critical points are metastable vacua.

In this work we reexamine the stability of de Sitter critical points in a general four-dimensional $\mathcal{N} = 1$ supergravity. Our tool is random matrix theory: $\mathcal{H}$ is a large matrix, and a great deal can be said about its eigenvalue spectrum. Moreover, given an ensemble of random matrices that typically have negative eigenvalues, the Tracy-Widom theory of fluctuations of extreme eigenvalues allows one to compute the probability of drawing a positive-definite matrix from the ensemble [143]. The key phenomenon is eigenvalue repulsion: a large fluctuation through which all eigenvalues become positive generally requires an increase in the local eigenvalue density, which is statistically costly.

We obtain results that depend on the relative sizes of the soft supersymmetry-breaking masses and the supersymmetric masses. At a generic critical point, the supersymmetric masses are not hierarchically larger than the soft masses, and supersymmetry provides negligible protection from instability. We develop a random matrix model for $\mathcal{H}$ and obtain an analytic expression for its eigenvalue spectrum. The spectrum has considerable support at negative values, so that tachyons are generic. Building on extensions of the Tracy-Widom theory due to Dean and Majumdar [39, 40], we then argue that the probability P of a fluctuation rendering $\mathcal{H}$ positive-definite is

$$P \propto \exp(-c\,N^{p})\,, \tag{4.1}$$

at leading order in large N, where N is the number of complex scalar fields, and c, p are constants, with $p \approx 1.5$. This is qualitatively similar to the original results of Aazami and Easther [144], who obtained $p \approx 2$ in a simpler model of the Hessian matrix. We conclude that an exceedingly small fraction of generic critical points are metastable.

The more promising regime, as stressed by Denef and Douglas, is that in which approximate supersymmetry protects against instabilities. When the soft masses are small compared to the supersymmetric masses, the only significant risk of an instability arises from the direction parameterized by the scalar superpartner of the Goldstino,[1] and we show that at a critical point that is generic apart from this requirement of approximate supersymmetry, there are almost always two tachyons. We identify a negative-definite contribution to these eigenvalues that is a manifestation of eigenvalue repulsion between the Goldstino directions and the supersymmetrically-stabilized scalars. The eigenvalue repulsion contribution is dominant at large N, so that it is extremely improbable that both eigenvalues fluctuate to become positive: the probability of positivity again takes the form (4.1), but now with $p \approx 1.3$.

Although our results clearly demonstrate that an exponentially small fraction of the critical points of a generic random supergravity theory are metastable, this finding in no way precludes the existence of a landscape of metastable de Sitter vacua. First of all, the actual number of metastable vacua can be extremely large – and in particular, larger than 10^{120} – while still being exponentially small compared to the number of critical points. Second, our analysis applies when K and W are generic functions of all of their arguments. A well-motivated configuration violating this assumption is a theory involving

[1] For extensive investigations of this unstable direction, see [145].

two decoupled sectors, containing N_H and N_L scalar fields, respectively. If the N_H fields receive large supersymmetric masses and dynamical supersymmetry breaking occurs at a lower scale in the sector containing N_L fields, then the dominant factor in the number of critical points can be exponential in N_H, while the fraction of critical points that are unstable is proportional to $\exp(-cN_L^p)$. Thus, for $N_H \gg N_L$ our findings yield only a modest reduction in the number of vacua.

The organization of this Chapter is as follows. In §4.2 we set our notation and review the structure of the Hessian matrix $\mathcal{H}$ at a critical point in a general four-dimensional $\mathcal{N} = 1$ supergravity, following [33]. In §4.3 we introduce the ideas from random matrix theory that are essential in this work, reviewing the relevant ensembles and assembling results from the theory of fluctuations of extreme eigenvalues. In §4.4 we apply these methods to study $\mathcal{H}$ at a generic critical point. We compute the eigenvalue spectrum analytically and obtain the probability of a large fluctuation that renders $\mathcal{H}$ positive-definite. In §4.5 we study $\mathcal{H}$ at an approximately-supersymmetric critical point. We show that eigenvalue repulsion typically leads to two negative eigenvalues associated to the Goldstino, and we again compute the probability of a fluctuation to positivity. In §4.6 we discuss extensions of our assumptions, and we illustrate our results in the KKLT scenario. We conclude in §4.7.

4.2 Critical Points in $\mathcal{N} = 1$ Supergravity

In this section we will discuss the form of the critical point equation, and describe the structure of $\mathcal{H}$ at a critical point, in a general four-dimensional $\mathcal{N} = 1$ supergravity. In §4.2.1 and §4.2.2 we closely follow the work of Denef and Douglas [33], reviewing how the critical point equation can be written as an eigenvalue equation, and we establish notation for the different contributions to the Hessian matrix. Then, in §4.2.3, we give a precise definition of a random supergravity, whose critical points will be the object of study in §4.4 and §4.5.

The F-term potential in an $\mathcal{N} = 1$ supergravity with N chiral superfields is given by

$$V = e^K \left(F_a \bar{F}^a - 3|W|^2 \right) , \tag{4.2}$$

with $a = 1, \ldots N$, in units in which $8\pi G \equiv M_{Pl}^{-2} = 1$. Here $\bar{F}^a = K^{a\bar{b}} \bar{F}_{\bar{b}} = K^{a\bar{b}} \bar{D}_{\bar{b}} \overline{W}$, where D_a is the Kähler covariant derivative, $D_a W = \partial_a W + K_a W$, and $K_{a\bar{b}}$ is the Kähler metric. We will use the shorthand $F_a \bar{F}^a \equiv F^2$.

We consider a set of critical points $\{q\}$, satisfying

$$\partial_a V|_q = e^K \Big(\mathcal{D}_a(F_b) \bar{F}^b - 2 F_a \overline{W} \Big) = 0\,. \tag{4.3}$$

Here and henceforth $\mathcal{D}_a$ denotes the appropriate Kähler and geometrically covariant derivative. At any given point q the scalar potential can be simplified by specifying the Kähler gauge such that $\langle K \rangle_q = 0$ and performing an appropriate coordinate transformation such that $K_{a\bar{b}}|_q = \delta_{a\bar{b}}$.

4.2.1 Matrix form of the critical point equation

The critical point equation (4.3) can be written as an eigenvalue equation of a particular Hermitian matrix, $\mathcal{M}$, formed from the second covariant derivatives of the superpotential [33]. Defining

$$Z_{ab} \equiv \mathcal{D}_a F_b \,, \tag{4.4}$$

and with $\vartheta_W \equiv \mathrm{Arg}(W)$, equation (4.3) can be expressed as

$$\mathcal{M}\hat{F} = 2|W|\hat{F}\,, \tag{4.5}$$

where

$$\mathcal{M} = \begin{pmatrix} 0 & e^{-i\vartheta_W} Z_{ab} \\ e^{i\vartheta_W} \bar{Z}_{\bar{a}\bar{b}} & 0 \end{pmatrix}, \tag{4.6}$$

and the $2N$-dimensional vector $\hat{F}$ is given by

$$\hat{F} = \begin{pmatrix} e^{-i\vartheta_W} F^{\bar{b}} \\ e^{i\vartheta_W} \bar{F}^{b} \end{pmatrix}. \tag{4.7}$$

The eigenvalues of $\mathcal{M}$ come in real pairs with opposite signs, $\pm\lambda_a$, with $\lambda_a \geq 0$ and $a = 1, \ldots, N$. Thus, at any critical point, $\mathcal{M}$ must have an eigenvalue equal to $2|W|$, and the vector $\hat{F}$ must be proportional to the corresponding eigenvector. In §4.3.1 we will discuss the spectrum of eigenvalues of $\mathcal{M}$.

4.2.2 Structure of the Hessian matrix

For a critical point q to be a metastable vacuum, the eigenvalues of the Hessian matrix of the scalar potential evaluated at q must all be positive. Denoting $U_{abc} = \mathcal{D}_a\mathcal{D}_b F_c$, and writing the curvature of the field space[2] in terms of the partial derivatives of the Kähler potential as $R_{a\bar{b}c\bar{d}} = \delta_{a\bar{f}}\,\partial_c \bar{\Gamma}^{\bar{f}}_{\bar{b}\bar{d}} = K_{a\bar{b}c\bar{d}} - K_{ac}{}^{e} K_{\bar{b}\bar{d}e}$, the bosonic mass matrix can be written

$$\begin{aligned} \partial^2_{ab} V &= U_{abc}\bar{F}^c - \overline{W} Z_{ab}\,, & (4.8)\\ \partial^2_{a\bar{b}} V &= \delta_{a\bar{b}}\Big(F^2 - 2|W|^2\Big) - F_a \bar{F}_{\bar{b}} - R_{a\bar{b}c\bar{d}} \bar{F}^c F^{\bar{d}} + Z_a^{\;\bar{c}}\, \bar{Z}_{\bar{b}\bar{c}}\,, & (4.9) \end{aligned}$$

[2]Our sign convention differs from that of [33].

where indices are raised and lowered with $\delta^{a\bar{b}}$. The Hessian matrix $\mathcal{H}$ is thus given by

$$\mathcal{H} = \begin{pmatrix} \partial^2_{a\bar{b}}V & \partial^2_{ab}V \\ \partial^2_{\bar{a}\bar{b}}V & \partial^2_{\bar{a}b}V \end{pmatrix} \tag{4.10}$$

$$= \begin{pmatrix} Z_a{}^{\bar{c}}\,\bar{Z}_{\bar{b}\bar{c}} - F_a\bar{F}_{\bar{b}} - R_{a\bar{b}c\bar{d}}\bar{F}^c F^{\bar{d}} & U_{abc}\bar{F}^c - Z_{ab}\overline{W} \\ \overline{U}_{\bar{a}\bar{b}\bar{c}}F^{\bar{c}} - \bar{Z}_{\bar{a}\bar{b}}W & \bar{Z}_{\bar{a}}{}^{c}\,Z_{bc} - F_b\bar{F}_{\bar{a}} - R_{b\bar{a}c\bar{d}}\bar{F}^c F^{\bar{d}} \end{pmatrix} +$$

$$+ \ \mathbb{1}\Big(F^2 - 2|W|^2\Big), \tag{4.11}$$

where $\mathbb{1}$ denotes the $2N \times 2N$ unit matrix. The Hessian matrix is most conveniently analyzed in a 'Goldstino' basis in which $F_a = \delta_a{}^1 F e^{i\vartheta_F}$. In this basis the critical point equation (4.5) can be written

$$Z_{11} = 2|W|\,e^{i(2\vartheta_F - \vartheta_W)}\,, \qquad Z_{1a'} = 0\,, \tag{4.12}$$

while the components $Z_{a'b'}$ remain unconstrained for $a', b' = 2, \ldots, N$. The Hessian matrix can be decomposed into constituent matrices as follows:

$$\mathcal{H} = \mathcal{H}_{\rm susy} + \mathcal{H}_{\rm pure} + \mathcal{H}_{K^{(4)}} + \mathcal{H}_{K^{(3)}} + \mathcal{H}_{\rm shift}\,, \tag{4.13}$$

where

$$\mathcal{H}_{\rm susy} = \begin{pmatrix} Z_a{}^{\bar{c}}\,\bar{Z}_{\bar{b}\bar{c}} & 0 \\ 0 & \bar{Z}_{\bar{a}}{}^{c}\,Z_{bc} \end{pmatrix}, \tag{4.14}$$

$$\mathcal{H}_{\rm pure} = \begin{pmatrix} 0 & U_{ab1}\bar{F}^1 - Z_{ab}\overline{W} \\ \overline{U}_{\bar{a}\bar{b}\bar{1}}F^{\bar{1}} - \bar{Z}_{\bar{a}\bar{b}}W & 0 \end{pmatrix}, \tag{4.15}$$

$$\mathcal{H}_{K^{(4)}} = F^2 \begin{pmatrix} -K_{a\bar{b}1\bar{1}} & 0 \\ 0 & -K_{b\bar{a}1\bar{1}} \end{pmatrix}, \tag{4.16}$$

$$\mathcal{H}_{K^{(3)}} = F^2 \begin{pmatrix} K_{a1}{}^{e}\,K_{\bar{b}\bar{1}e} & 0 \\ 0 & K_{\bar{a}\bar{1}}{}^{\bar{e}}\,K_{b1\bar{e}} \end{pmatrix}, \tag{4.17}$$

$$\mathcal{H}_{\rm shift} = \mathbb{1}\Big(F^2 - 2|W|^2\Big) - F^2\delta_a{}^1\delta_{\bar{b}}{}^{\bar{1}} - F^2\delta_{\bar{a}}{}^{\bar{1}}\delta_b{}^1\,. \tag{4.18}$$

A few remarks are appropriate at this point. First, $\mathcal{H}_{\text{susy}}$ and $\mathcal{H}_{K^{(3)}}$ are positive semidefinite. Second, the mass scale m_{susy} of the supersymmetric masses is set by the eigenvalues of $\mathcal{H}_{\text{susy}}$, and can be larger or smaller than the scale F that determines the soft supersymmetry-breaking masses. The ratio F/m_{susy} (recall that we have set $M_{Pl} = 1$) has a significant effect on stability. In §4.4 we will study generic critical points, at which[3] $F \sim m_{\text{susy}}$, and in §4.5 we will consider 'approximately-supersymmetric' critical points at which $F \ll m_{\text{susy}}$.

In §4.3, we will explain how the constituent matrices of $\mathcal{H}$ given in equations (4.14)-(4.18) can be identified as — or well-approximated by — elements of classical random matrix ensembles with well-known emergent eigenvalue spectra at large N. The distribution of the eigenvalues of the Hessian matrix can then be obtained as the free convolution [146] of the eigenvalue distributions of the constituent matrices, just as the distribution of a scalar random variable that is the sum of terms with known distributions can be obtained by the ordinary convolution of the constituent probability density functions.

4.2.3 Defining a random supergravity

To proceed further, we must specify the statistical properties of the entries of the matrices (4.14)-(4.18) constituting the Hessian matrix $\mathcal{H}$. Our fundamental assumption — consistent with that of [33] — is that the components of tensors formed by covariant differentiation of W and K are *independent, identically distributed (i.i.d.) variables* drawn from some statistical distribution Ω. We will occasionally abbreviate this by saying that W and K are *random functions*. Note that this assumption is quite different from taking the entries of $\mathcal{H}$ itself to be

[3] See Appendix B.1 for a discussion of the distribution of F/m_{susy} in the set of all critical points.

i.i.d. variables drawn from a distribution Ω, which omits the structure and correlations implicit in (4.13).

We will use $\Omega(\mu, \sigma)$ to denote a complex[4] distribution whose magnitude has mean μ and standard deviation σ, with a uniform distribution for the phase. In §4.6.1 we explain that as a consequence of the well-known phenomenon of universality in random matrix theory, the precise choice of Ω is immaterial, provided that the higher moments of Ω are appropriately bounded.

The Kähler potential and its derivatives

Suppose we took the Kähler potential to be a random function such that in a generic coordinate basis,

$$K_{a\bar{b}}|_q \in \Omega(0, 1)\,. \tag{4.19}$$

Performing an orthogonal rotation to diagonalize $K_{a\bar{b}}|_q$, the resulting eigenvalues will generically be of order N (see §4.3 for details), and the $GL(N, \mathbb{C})$ transformation required to achieve $K_{a\bar{b}} = \delta_{a\bar{b}}$ involves rescaling by factors of order N. To avoid performing this rescaling in all terms involving K, we find it convenient to take

$$K_{a\bar{b}}|_q \in \Omega(0, \tfrac{1}{\sqrt{N}})\,. \tag{4.20}$$

Then, the $GL(N, \mathbb{C})$ transformation leading to $K_{a\bar{b}} = \delta_{a\bar{b}}$ does not involve any N-dependent rescalings. More generally, the choice $\Omega(0, \frac{1}{\sqrt{N}})$ is convenient because the random matrix eigenvalue spectra presented in §4.3 then have support in the same domain for all N.

Next, we need to specify the properties of $K_{a\bar{b}c}$ and $K_{a\bar{b}c\bar{d}}$ at q, in the basis

[4] The diagonal elements of Hermitian matrices will of course be real.

in which $K_{a\bar{b}} = \delta_{a\bar{b}}$. Ideally, the statistics of these objects would follow from a theory of general Kähler metrics (see e.g. [147] for a very recent discussion of related issues), but for our purposes it will suffice to stipulate that the Kähler potential K is a random function of its arguments, in the sense described above. Then, $K_{a\bar{b}c}$ and $K_{a\bar{b}c\bar{d}}$ do not take a special form in the basis in which the metric is diagonalized, and imposing the critical point equation (4.5) does not change this situation. In the 'Goldstino' basis in which $F_a = \delta_a^{\ 1} F\, e^{i\vartheta_F}$ at q, we have

$$K_{a\bar{b}}|_q = \delta_{a\bar{b}}\,, \tag{4.21}$$

$$K_{a\bar{b}1}|_q \in \Omega(0, \tfrac{1}{\sqrt{N}})\,, \tag{4.22}$$

$$K_{a\bar{b}1\bar{1}}|_q \in \Omega(0, \tfrac{1}{\sqrt{N}})\,. \tag{4.23}$$

The assumptions (4.22),(4.23) are well-motivated for general Kähler manifolds, but we note that there are interesting exceptions, including the special geometry of the vector multiplet moduli space in $\mathcal{N} = 2$ supergravity, for which the curvature tensor is given by

$$R_{a\bar{b}c\bar{d}} = K_{a\bar{b}c\bar{d}} - K_{ac}^{\ \ e} K_{\bar{b}\bar{d}e} = K_{a\bar{b}} K_{c\bar{d}} + K_{a\bar{d}} K_{c\bar{b}} - e^{2K} K^{p\bar{q}} \mathcal{F}_{acp} \bar{\mathcal{F}}_{\bar{b}\bar{d}\bar{q}}\,, \tag{4.24}$$

where $\mathcal{F}$ is the prepotential. Repeating the analyses of §§4.4,4.5 with the special geometry relationship (4.24) is straightforward, and we find a *decreased* likelihood of stability compared to the more general assumptions (4.22),(4.23) that are used throughout this work.

The superpotential and its derivatives

Turning now to the superpotential and its derivatives, we begin with a warmup in global supersymmetry. Fixing a point q in field space and working with canonically-normalized fields ϕ_A, $A = 1, \ldots N$, we may write W in the form (momentarily restoring factors of the Planck mass for clarity)

$$W = M^3\, w(\phi_1/M_{Pl}, \ldots, \phi_N/M_{Pl})\,, \tag{4.25}$$

where M is a mass scale. Our assumption is that w is a random function of its dimensionless arguments $x_A \equiv \phi_A/M_{Pl}$, so that

$$\frac{\partial w}{\partial x_A}\Big|_q \in \Omega(\mu, \sigma)\,, \qquad \frac{\partial^2 w}{\partial x_A \partial x_B}\Big|_q \in \Omega(\mu, \sigma)\,, \tag{4.26}$$

etc. At a *typical* point the various derivatives of the superpotential will be of the same order of magnitude, set by the physical effect responsible for the superpotential. (For example, in type IIB flux compactifications, the sizes of the superpotential and its derivatives are set by the flux scale.) However, *atypical* points will play an important role in §4.5: it can happen that the superpotential, as well as its first derivative F_a, are small compared to higher derivatives. The result is a significant change in the stability criteria [33].

In supergravity, the relevant expansion around q is in (Kähler and geometrically) covariant derivatives of the superpotential. We take

$$F_a \equiv \mathcal{D}_a W \quad \in \quad m_{\rm susy}\, \Omega(0, \tfrac{1}{\sqrt{N}})\,, \tag{4.27}$$

$$Z_{ab} \equiv \mathcal{D}_a \mathcal{D}_b W \quad \in \quad m_{\rm susy}\, \Omega(0, \tfrac{1}{\sqrt{N}})\,, \tag{4.28}$$

$$U_{abc} \equiv \mathcal{D}_a \mathcal{D}_b \mathcal{D}_c W \quad \in \quad m_{\rm susy}\, \Omega(0, \tfrac{1}{\sqrt{N}})\,. \tag{4.29}$$

We will assume that supersymmetry is spontaneously broken by an F-term; for a discussion of the possible effects of D-term energy (cf. [33]), see §4.6.3. The requirement of nonnegative vacuum energy then becomes

$$F \geq \sqrt{3}|W|\,, \tag{4.30}$$

so that in particular, $|W| \lesssim O(F)$. It is useful to define

$$\omega^2 \equiv \frac{3|W|^2}{F^2}, \tag{4.31}$$

so that $V = F^2(1 - \omega^2)$, and equation (4.30) translates to $\omega \le 1$. The stability properties of critical points depend on ω, so we will repeat our analysis for a collection of fixed values of $\omega \in [0, 1]$. Thus, $|W|$ is determined in terms of F and ω. For more details on this point, see Appendix B.1.

We can now construct a model of a random supergravity by drawing F_a, Z_{ab}, U_{abc}, $K^{(3)}_{a\bar{b}c}$, and $K^{(4)}_{a\bar{b}c\bar{d}}$ independently from the distributions specified above, at each point q. As we are primarily interested in critical points, we should study the set of points $\{q\}$ subject to the critical point equation (4.5). Such points are *not* completely generic: equation (4.5) enforces a particular correlation between Z_{ab}, W, and F_a, as reviewed in the discussion following equation (4.5). Following [33], we will carefully incorporate the restriction implied by equation (4.5).

4.3 Random Matrix Theory for Supergravity

In this section we will briefly review a few important concepts and results from random matrix theory, in order to make our analysis more self-contained. An accessible and fairly recent introduction can be found in [148]; see also the text by Mehta [149].

4.3.1 Classical ensembles

A foundational idea in random matrix theory is that given only limited statistical information about the entries of a diagonalizable $N \times N$ matrix, for large N one can make incisive statements about the statistical properties of the eigenvalues. For our purposes, the properties of principal interest are the eigenvalue spectrum for a typical matrix, and the probability of a large fluctuation of the smallest eigenvalue.

We begin by reviewing the ensembles relevant for this work.

The Wigner ensemble

One of the simplest and best-known ensembles of random matrices is the *Wigner ensemble* of Hermitian matrices, also referred to as the Gaussian Unitary Ensemble [150, 151, 152]. Elements of this ensemble, which we refer to as Wigner matrices, are $N \times N$ Hermitian matrices M given by

$$M = A + A^{\dagger}\,, \tag{4.32}$$

where A_{ij} for $i, j = 1, \ldots, N$ are i.i.d. variables drawn from $\Omega(0, \sigma)$, and the dagger denotes Hermitian conjugation.

The measure on the space of matrices is

$$dP(M) = \prod_{1 \le i \le j \le N} f(M_{ij})\, dM_{ij}\,, \tag{4.33}$$

where $f(M_{ij})$ denotes the probability density of observing M_{ij}. For normally-distributed entries of M, the joint probability density of the eigenvalues

$\lambda_1, \ldots, \lambda_N$ is obtained by a unitary change of coordinates,

$$f(\lambda_1, \ldots, \lambda_N) = C \exp\Big(-\frac{1}{\sigma^2}\sum_{i=1}^{N} \lambda_i^2 + 2\sum_{i<j}^{N} \ln|\lambda_i - \lambda_j|\Big), \tag{4.34}$$

where C is an N-dependent normalization constant. As conceived in the famous work of Dyson [153], this joint probability density can be given a physical interpretation in terms of a one-dimensional Coulomb gas of N charged particles executing Brownian motion under the influences of a confining quadratic potential and of mutual electrostatic repulsion. This physical picture has proved to be very fruitful in deriving exact results for a variety of properties of the eigenvalue spectrum (see e.g. [39, 40]), and in §4.4 and §4.5 we will see that repulsion between pairs of eigenvalues significantly impacts the stability of critical points in supergravity.

At large N, the eigenvalue spectrum of a Wigner matrix converges to the celebrated Wigner semicircle law,

$$\rho(\lambda) = \frac{1}{2\pi N\sigma^2}\sqrt{4N\sigma^2 - \lambda^2}\,. \tag{4.35}$$

where ρ is the eigenvalue density. Setting $\sigma = \frac{1}{\sqrt{N}}$, the eigenvalue spectrum has support in the interval $[-2, 2]$, cf. Figure 4.1.

The Wishart ensemble

The second class of random matrices we will need are *complex Wishart matrices*, which take the form

$$M = AA^{\dagger}\,, \tag{4.36}$$

where A is an $N \times Q$ complex matrix with entries drawn from $\Omega(0, \sigma)$, and $Q \geq N$. The study of this ensemble dates back to Wishart's investigation of sample covariance matrices [154], and the universality evident in the Wishart ensemble

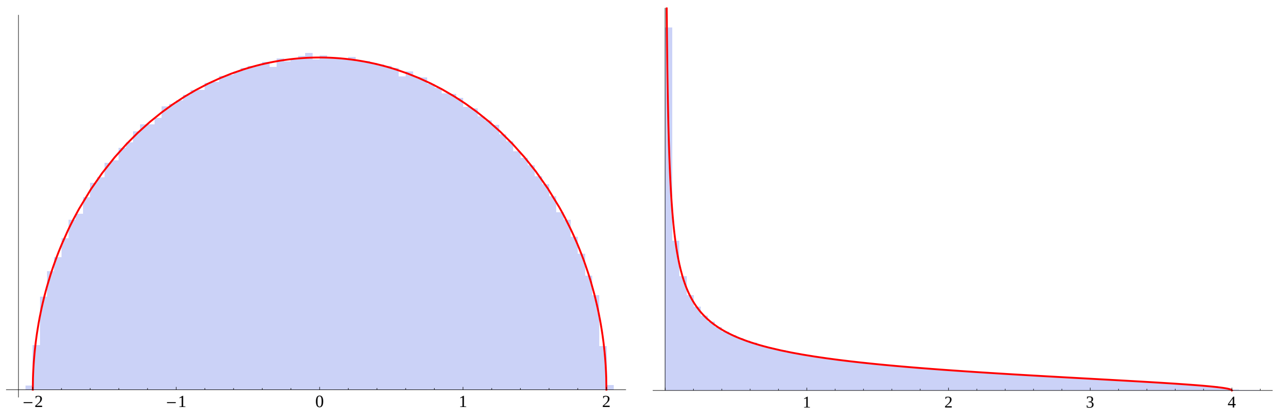


Figure 4.1: The eigenvalue spectra for the Wigner ensemble (left panel), and the Wishart ensemble with $N = Q$ (right panel), from 10^3 trials with $N = 200$.

provided some of the inspiration for Wigner's subsequent development of random matrix theory.

As a Wishart matrix is the Hermitian square of another matrix, it is necessarily positive semidefinite. The joint probability density of a complex Wishart matrix is (cf. e.g. [148])

$$f(\lambda_1, \ldots, \lambda_N) = C \, \exp\Big(-\frac{1}{\sigma} \sum_{i=1}^{N} \lambda_i \; + 2 \sum_{i<j}^{N} \ln|\lambda_i - \lambda_j| + (Q - N) \sum_{i}^{N} \ln\lambda_i \Big). \quad (4.37)$$

In the Coulomb gas picture, the non-negativity of a Wishart matrix corresponds to the presence of a hard wall at $\lambda = 0$.

The eigenvalue distribution in the Wishart ensemble is given by the Marčenko-Pastur law [156], which takes the form

$$\rho(\lambda) = \frac{1}{2\pi N\sigma^2 \lambda} \sqrt{(4N\sigma^2 - \lambda)\lambda}\,, \quad (4.38)$$

for the special case $N = Q$ that will be relevant in our analysis, cf. Figure 4.1.

The probability density function of the smallest eigenvalue λ_1 was first computed by Edelman [155], and for our purposes it suffices to note that for $N = Q$ and $\sigma = \frac{1}{\sqrt{N}}$, its average position $\langle \lambda_1 \rangle$ scales as $\frac{1}{N^2}$.

The Altland-Zirnbauer CI ensemble

The matrix $\mathcal{M}$ appearing in the critical point equation (4.5) has an eigenvalue spectrum that is broadly reminiscent of the Wigner semicircle law, but the $2N$ eigenvalues of $\mathcal{M}$ come in opposite-sign pairs $\pm\lambda_a$, with $0 \leq \lambda_1 \leq \ldots \leq \lambda_N$. As observed in [33], matrices $\mathcal{M}$ of the form (4.6) belong to the *Altland-Zirnbauer CI ensemble* [157]. For normally-distributed entries of $\mathcal{M}$, the joint probability density of the eigenvalues is

$$f(\lambda_1, \ldots, \lambda_N) = C \exp\Big(-\frac{1}{\sigma^2}\sum_{i=1}^{N}\lambda_i^2 + \sum_{i\neq j}^{N}\ln|\lambda_i^2 - \lambda_j^2| + \sum_{i=1}^{N}\ln|\lambda_i|\Big). \tag{4.39}$$

In the Coulomb gas picture, the additional term $\sum_{i=1}^{N}|\lambda_i|$ can be interpreted as encoding a repulsive force between each mirror pair of eigenvalues, $\pm\lambda_i$ [157]. This repulsion is particularly important for the smallest eigenvalue of $\mathcal{M}$, and leads to a linear cleft in the eigenvalue spectrum for small λ:

$$\rho(\lambda) \approx k\lambda + O(\lambda^3), \tag{4.40}$$

with k a constant of order unity, so that the eigenvalue density vanishes at $\lambda = 0$, cf. Figure 4.2. Recalling that the critical point equation (4.5) requires that $\mathcal{M}$ has $2|W|$ as an eigenvalue, we see that critical points with very small $|W|$ are rare in comparison to those with $2|W| \approx 1$.

4.3.2 Fluctuations of extreme eigenvalues

The eigenvalue spectra presented above describe the *typical* configurations of eigenvalues: for example, the eigenvalue spectrum for an ensemble of Wigner matrices with entries drawn from $\Omega(0, \sigma)$ is zero outside $[-2\sqrt{N}\sigma, 2\sqrt{N}\sigma]$, but

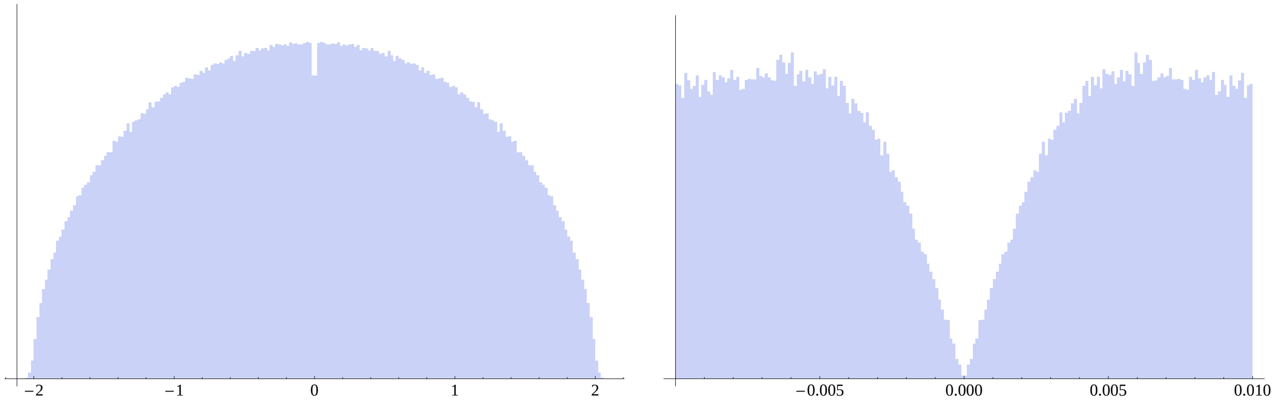


Figure 4.2: The eigenvalue spectrum for the CI ensemble, from 10^5 trials with $N = 200$. The full spectrum appears in the left panel, while the right panel shows the details of the cleft at $\lambda = 0$. Notice that the boundary of the linear regime occurs for $\lambda \sim \frac{1}{N}$.

this does not mean that no matrix in the ensemble has eigenvalues outside this range. Instead, the Wigner spectrum has a 'soft edge' at each end of the semicircle: there is a nonzero probability that one or more eigenvalues can be found beyond this edge. In contrast, the Wishart spectrum has a 'hard edge' at $\lambda = 0$, as the matrices in question are necessarily positive semidefinite, while the other edge of the spectrum is soft.

As we shall soon establish, the spectrum of the Hessian matrix $\mathcal{H}$ extends to negative values, so that the presence of tachyons is generic, but not guaranteed. It is therefore essential to determine the probability that the smallest eigenvalue of $\mathcal{H}$ happens to be large enough so that $\mathcal{H}$ is positive definite. We refer to this occurrence as a fluctuation to positivity.

Eigenvalue fluctuations and the Tracy-Widom law

The study of fluctuations of the smallest (or largest) eigenvalue was initiated in the pioneering work of Tracy and Widom [143]; see also [158]. The theory of fluctuations is best-developed when the fluctuations are suitably small, with the deviation of the extreme eigenvalue from its mean position being $O(N^{-1/6})$ for the case of the Wigner ensemble. In this case, the smallest eigenvalue λ_1 is given by (cf. [159] for a useful summary)

$$\lambda_1 \approx -2\sqrt{N} + N^{-1/6}\chi\,, \tag{4.41}$$

where for $N \to \infty$, χ is a random variable that follows the Tracy-Widom distribution F_2 [143]. Extension of the Tracy-Widom law to the Wishart ensemble was achieved for real matrices in [160], and for complex Wishart matrices in [161].

In §4.4 we will find that for a typical supergravity critical point, a fluctuation of size $O(N^{-1/6})$ of the smallest eigenvalue of $\mathcal{H}$ is insufficient to render $\mathcal{H}$ positive definite. We therefore require an extension of the Tracy-Widom theory describing large fluctuations, with the deviation from the mean position being as large as $O(\sqrt{N})$. The theory of large fluctuations has been developed[5] in a series of works by Majumdar and collaborators [39, 164, 40, 165, 166, 159], which we now briefly review, focusing on the Wigner ensemble.

Through a saddle point computation of the partition function in the Coulomb gas model, Dean and Majumdar [39, 40] were able to evaluate the probability of a large fluctuation of the smallest eigenvalue λ_1 of a Wigner matrix to the right of its mean position $\langle\lambda_1\rangle \equiv -2\sqrt{N}$. The result, at leading order in

[5]Earlier work on related fluctuations appears in [162]. For applications to counting critical points of random functions, see e.g. [163].

large N, is [39, 40]

$$P\Big(\lambda_1 \geq t\Big) \propto \exp\Big[-2N^2\psi_-\left(t/\sqrt{2N}\right)\Big], \tag{4.42}$$

for $t \geq -2\sqrt{N}$ and $t + 2\sqrt{N} \sim O(\sqrt{N})$. Although $\psi_-(y)$ is known in closed form, we present here only the result relevant for a fluctuation to positivity:

$$\psi_-(0) = \frac{\ln(3)}{4}. \tag{4.43}$$

(The corresponding result of [39, 40] for *real* Wigner matrices agrees very well with the earlier numerical results of Aazami and Easther [144] for the same ensemble.) In summary, the probability that an $N \times N$ complex Wigner matrix is positive-definite is given by [144, 39, 40]

$$P \propto \exp\Big[-cN^2\Big], \tag{4.44}$$

with $c \approx \frac{\ln(3)}{2}$. At large N this is exceptionally small compared to the estimate $P \approx 2^{-N}$ that follows from the naive assumption that the N eigenvalues are independent. Of course, eigenvalue interactions are fundamental to random matrix theory, so it is no surprise that omitting these interactions gives an entirely inaccurate result for the probability of positivity.

An intuition from the Coulomb gas model will be helpful in our analysis. Consider a fluctuation of the smallest eigenvalue λ_1 to roughly the midpoint of the spectrum, as would be required for a fluctuation to positivity in the Wigner ensemble. The distance involved is $O(\sqrt{N})$, and $O(N)$ eigenvalues need to be displaced. As these eigenvalues experience a quadratic potential, the total energetic cost is $O(N^2)$, consistent with the detailed results of [39, 40]. Similar results have been obtained for inward fluctuations of the soft and hard edges of the Wishart spectrum in [164] and [167], respectively. The lesson is that a substantial inward shift of one or more eigenvalues has a statistical cost $\sim \exp(-N^2)$, and is hence extremely unlikely at large N.

Having assembled the necessary tools, we now turn to studying the stability of the Hessian.

4.4 Stability of Generic Critical Points

In this section we will study the Hessian $\mathcal{H}$ at a generic critical point, where

$$F_a \sim Z_{ab} \sim U_{abc} \,. \tag{4.45}$$

We refer the reader to Appendix B.1 for a detailed demonstration that such points are indeed generic.

In §4.4.1 we examine the decomposition (4.13) of the Hessian matrix into a sum of terms and argue that each term is well-approximated as a member of one of the classical ensembles reviewed in §4.3.1. We then obtain the eigenvalue spectrum analytically from the free convolution of the constituent spectra. In §4.4.2 we argue that the probability that a given critical point is a metastable vacuum can be obtained by adapting the results of [39, 40] to the free convolution model. We then perform an extensive numerical analysis of the full Hessian matrix, finding that a generic critical point is exponentially unlikely to be a metastable vacuum. Thus, despite the abundance of critical points, this region of the random supergravity landscape is indeed a wasteland.

4.4.1 The Hessian spectrum from a free convolution

The Hessian $\mathcal{H}$ can be decomposed according to (4.13) as $\mathcal{H} = \mathcal{H}_{\rm susy} + \mathcal{H}_{\rm pure} + \mathcal{H}_{K^{(4)}} + \mathcal{H}_{K^{(3)}} + \mathcal{H}_{\rm shift}$.

From §4.3 we immediately recognize that each of $\mathcal{H}_{\rm susy}$ and $\mathcal{H}_{K^{(3)}}$ is very similar to a double copy of an N-dimensional complex Wishart matrix. The correspondence is imperfect because Z (respectively $K^{(3)}$) is symmetric, so that the diagonal entries have twice the variance of the off-diagonal entries. We have verified that this minor difference does not significantly affect the eigenvalue spectrum.

Next, $\mathcal{H}_{\rm pure}$ and $\mathcal{H}_{K^{(4)}}$ can be modeled as $2N$-dimensional Wigner matrices. Once again, this is an approximation: the actual Hessian matrix at a critical point must incorporate the critical point constraint (4.5). The sum of two Wigner matrices is again a Wigner matrix, so we may write $\mathcal{H}_{\rm pure} + \mathcal{H}_{K^{(4)}} \approx$ Wigner.

Assembling the pieces, and noting that the effect of $\mathcal{H}_{\rm shift}$ on the bulk of the eigenvalue spectrum is simply a translation, our model amounts to

$$\mathcal{H} \approx \mathrm{Wigner}(\mathcal{H}_{\rm pure} + \mathcal{H}_{\rm K^{(4)}}) + \mathrm{Wishart}(\mathcal{H}_{\rm susy}) + \mathrm{Wishart}(\mathcal{H}_{\rm K^{(3)}})\,. \tag{4.46}$$

Free convolutions and sums of random matrices

To obtain the spectrum, we need to address a problem of the general form: "if A and B are random matrices with known eigenvalue spectra μ_A, μ_B, what is the spectrum μ_{A+B} of their sum A+B?" If A and B were to commute, μ_{A+B} would be the convolution of μ_A and μ_B, but there is no justification for this assumption in our case. The solution of the general problem is provided by Voiculescu's theory of free probability. We will describe here only the immediately relevant tools of free probability, referring the reader to the text [146] for details and references.

Given two ensembles A, B of random matrices, the *free convolution* $\boxplus$ is defined such that

$$\mu_A \boxplus \mu_B = \mu_{A+B}\,, \tag{4.47}$$

i.e. the free convolution of the spectra of the summands is the spectrum of the sum. Just as cumulants are additive under ordinary convolution of random variables, free cumulants can be defined with the same additivity property under the free convolution. In principle μ_{A+B} can be obtained from the R-transform, which is the generating function of the free cumulants [168], but inversion of the R-transform can be rather cumbersome. For the large class of *algebraic* random matrices [169], which includes sums of Wigner and Wishart matrices, a more efficient approach [156, 169] is to work with the Stieltjes transform [169].

The Stieltjes transform of a probability measure $\mathrm{d}\mu(x) = \rho(x)dx$ with support on a real interval I is defined by

$$m_\mu(z) = \int_I \frac{\mathrm{d}\mu(x)}{z - x}\,, \tag{4.48}$$

where $\mathrm{Im}(z) > 0$. Algebraic random matrices are random matrices for which $m_\mu(z)$ is the solution of a polynomial equation in m_μ and z, e.g. the Stieltjes transform of the Wigner density solves the equation

$$m_\mu^2 + a\, z m_\mu + a = 0\,, \tag{4.49}$$

where $a = (N\sigma^2)^{-1}$. The probability density is readily obtained from $m_\mu(z)$ using the Stieltjes-Perron inversion formula,

$$\rho(x) = \frac{1}{\pi} \lim_{\epsilon \to 0} \mathrm{Im}\; m_\mu(x + i\epsilon)\,. \tag{4.50}$$

Edelman and Rao have shown that the free convolution can be implemented efficiently through manipulations of polynomials involving the Stieltjes transform [169].

The spectrum as Wigner ⊞ Wishart ⊞ Wishart

In terms of the free convolution ⊞ defined in equation (4.47), we may write the eigenvalue spectrum $\rho(\mathcal{H})$ as

$$\rho(\mathcal{H}) \approx \rho(\text{Wigner}) \boxplus \rho(\text{Wishart}) \boxplus \rho(\text{Wishart})\,, \tag{4.51}$$

where $\rho(\text{Wigner})$ is given in equation (4.35) and $\rho(\text{Wishart})$ is given in equation (4.38). Obtaining the Stieltjes transforms of $\rho(\text{Wigner})$ and $\rho(\text{Wishart})$ and using the polynomial method of [169], we find the spectrum

$$\rho(\lambda) = \frac{3\omega^4 + 117 + 9\left(\omega^2 - 4\right)\lambda + 9\lambda^2 - \left(\frac{3}{2}\right)^{2/3} \psi(\lambda)^{2/3}}{2^{2/3}\, 3^{5/6}\, \pi\, \psi(\lambda)^{1/3}\, \left(\omega^2 + 6\right)}\,, \tag{4.52}$$

where $\omega = \frac{\sqrt{3}|W|}{F}$, $\psi(\lambda)$ is given by

$$\psi(\lambda) \;=\; 9\omega^4(\lambda + 1) + 27\omega^2\left(\lambda^2 - 5\lambda + 24\right) - \sqrt{3\tau} + 18\lambda^3 - 108\lambda^2 + 216\lambda + 1345, \tag{4.53}$$

and

$$\begin{aligned} \tau \;=\; & -\left(\omega^2 + 6\right)^2 \Big[81\lambda^4 + 162\lambda^3\left(\omega^2 - 7\right) + 9\lambda^2\left(13\omega^4 - 180\omega^2 + 621\right) + \\ & + \; 18\lambda\left(2\omega^6 - 35\omega^4 + 333\omega^2 - 630\right) + 4\omega^8 - 48\omega^6 + 873\omega^4 - 12636\omega^2 - 9396\Big]. \end{aligned} \tag{4.54}$$

This is one of our primary results. Figure 4.3 illustrates the remarkably good agreement between (4.52) and simulations of the full $\mathcal{H}$.

4.4.2 Eigenvalue fluctuations and de Sitter vacua

Although we now have an analytic result for the eigenvalue spectrum in the Wigner ⊞ Wishart ⊞ Wishart model, which gives an excellent approximation to the spectrum of $\mathcal{H}$ itself, computing the probability of a large fluctuation

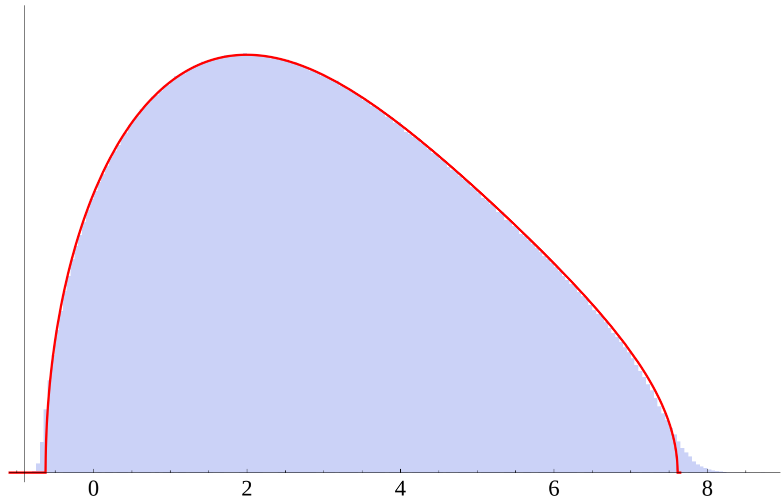


Figure 4.3: The histogram shows the spectrum of eigenvalues of the full Hessian matrix $\mathcal{H}$ (4.13) for $N = 200$ and $\omega = .1$, in units of F^2, while the curve gives the analytic result (4.52) from the Wigner $\boxplus$ Wishart $\boxplus$ Wishart model, with no adjustable parameters.

for this model is rather involved. The literature summarized in §4.3.2 contains detailed characterizations of the fluctuations of extreme eigenvalues of Wigner or Wishart matrices, but a direct computation of the large fluctuations in the Wigner $\boxplus$ Wishart $\boxplus$ Wishart model would require a dedicated saddle point analysis along the lines of [39, 40], and is beyond the scope of the present work.

From the Coulomb gas model, it is clear that a sufficiently large fluctuation of the smallest eigenvalue will be sensitive to the global shape of the spectrum: such a fluctuation will displace the eigenvalues to its right, with an energy cost that depends on their density. However, a small fluctuation will displace only the eigenvalues near the edge of the spectrum, and the likelihood of such a fluctuation should therefore depend only on the edge shape. Correspondingly, it has been conjectured [169] that Tracy-Widom fluctuations will be seen in essentially any algebraic random matrix whose eigenvalue density has square root behavior at its edge. As our Wigner $\boxplus$ Wishart $\boxplus$ Wishart model falls in this class, we expect that *small* fluctuations of the smallest eigenvalue of $\mathcal{H}$ will be governed by the Tracy-Widom law.

Examining the position of the left edge in the Wigner $\boxplus$ Wishart $\boxplus$ Wishart model, we see that a fluctuation to positivity is *not* a small fluctuation in the sense of [143]: the distance to the origin is[6] $O(\sqrt{N})$, not $O(N^{-1/6})$. However, between the left edge and the origin, the spectrum has a shape very reminiscent of the semicircle law. Emboldened by this, we anticipate that the probability of a large fluctuation of the smallest eigenvalue of $\mathcal{H}$ is accurately modeled using the corresponding probability (4.44) in the Wigner ensemble, i.e. we expect $P \propto \exp(-c\,N^p)$ with $p \sim 2$.

4.4.3 Numerical results

We now report on the results of extensive simulations of fluctuations to positivity in the full $\mathcal{H}$ model. These simulations make no approximation. We include the full structure of $\mathcal{H}$ (e.g., the slight difference between $\mathcal{H}_{\mathrm{susy}}$ and a Wishart matrix), and we do not rely on any of the analytical results reviewed in §4.3. No expansion in $\frac{1}{N}$ or in F is used. We simply create an ensemble of realizations of $\mathcal{H}$, following the prescription of §4.2, and directly determine the fraction that are positive definite.

Naturally, these simulations could still fail to yield an accurate picture of the positivity probability in the supergravities derived from string theory: in particular, our definition of a random supergravity could be non-representative. Moreover, computational cost imposes an upper limit on N, and our extrapola-

[6]For ease of comparison to [143], we take $\Omega = \mathcal{N}(0,1)$ in this discussion, although we set $\Omega = \mathcal{N}(0, \frac{1}{\sqrt{N}})$ elsewhere.

tion to larger N could be inaccurate.

One detail of $\mathcal{H}$ requires further explanation. The critical point condition, equation (4.5), enforces that $\mathcal{M}$ has eigenvalue $2|W|$. For a given W, no matrix drawn randomly will have a precisely correct eigenvalue (this reflects the fact that critical points are a measure zero subset of all points). To impose this constraint, we note that if $\mathcal{M}$ has an eigenvalue $\lambda_W \in [(2-2\epsilon)|W|, (2+2\epsilon)|W|]$, but $\lambda_W \neq 2|W|$, the distortions of the spectrum compared to that found at a genuine critical point will be of order ϵ. In our simulations, we have taken $\epsilon = 10^{-2}$. In §4.5 this issue will pose a greater challenge: for $|W| \ll 1/N$, a very small fraction of randomly drawn $\mathcal{M}$ will fall in $[(2-2\epsilon)|W|, (2+2\epsilon)|W|]$, cf. equation (4.40), and it becomes computationally costly to find examples.

Figure 4.4 presents the results of simulations of the full Hessian matrix $\mathcal{H}$ (upper curve) and of the Wigner $\boxplus$ Wishart $\boxplus$ Wishart model (lower curve). The qualitative properties are similar, but the best-fit values of p are somewhat different. This is not surprising, as the numerically-accessible values of N are not large: for $N \gtrsim 20$, stability is extremely rare, and it is difficult to obtain sufficient statistics to characterize the probability of positivity. For $N \lesssim 20$, the large N expansion underpinning our random matrix theory approach is marginal at best, with two important consequences. First, the correspondence between the model and simulation spectra is imperfect for $N \sim 20$ (contrast the superb agreement for $N = 200$ shown in Figure 4.3), and the corresponding difference between the left edges of these spectra contributes to a different fluctuation probability. A primary cause of the difference between the spectra of the the full Hessian matrix $\mathcal{H}$ and the analytical model (4.52) is that $\mathcal{H}_{\mathrm{pure}}$ involves the matrix Z, as does $\mathcal{H}_{\mathrm{susy}}$, so that in the Wigner $\boxplus$ Wishart $\boxplus$ Wishart model, the Wigner matrix is

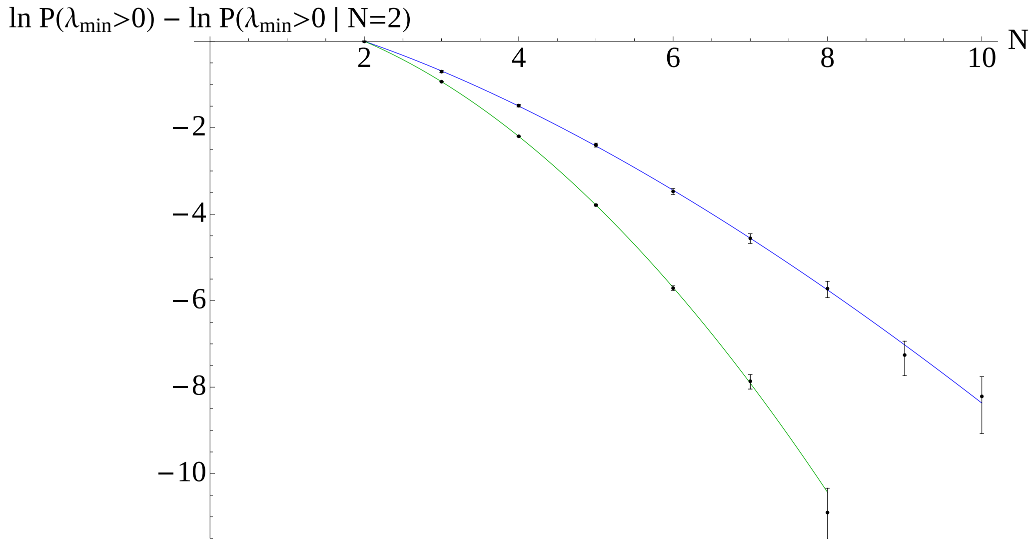


Figure 4.4: The logarithm of the probability $P(\lambda_{\min} > 0)$ that the smallest eigenvalue of $\mathcal{H}$ is positive, as a function of N, with $\omega = 1$. Upper branch: simulations of the full Hessian matrix $\mathcal{H}$, with best-fit values $p = 1.50 \pm 0.10$, $c = 0.29 \pm 0.06$. Lower branch: simulations of the Wigner ⊞ Wishart ⊞ Wishart model, with best-fit values $p = 1.90 \pm 0.04$, $c = 0.21 \pm 0.02$. The error bars give the 2σ statistical uncertainty.

correlated with one of the Wishart matrices. For $N \gg 1$ (and also for $\omega \ll 1$) this correlation becomes less important. Second, for small enough N, fluctuations to positivity are governed by the Tracy-Widom law (4.41) rather than by the considerably steeper large-fluctuation expression (4.42), so that a fit that includes data points starting from $N = 2$ will result in a value of p that is *smaller* than the asymptotic large N value.

In summary, due to the challenges inherent in studying extremely rare events numerically, we have not obtained sufficient data at large N to make a definitive determination of the large N behavior of the probability, and this is an interesting problem for the future. In light of the arguments of §4.4.2, it remains reasonable to conjecture that $p \sim 2$ at sufficiently large N.

Next, Figure 4.5 shows the trends in c and p, cf. equation (4.1), as ω is var-

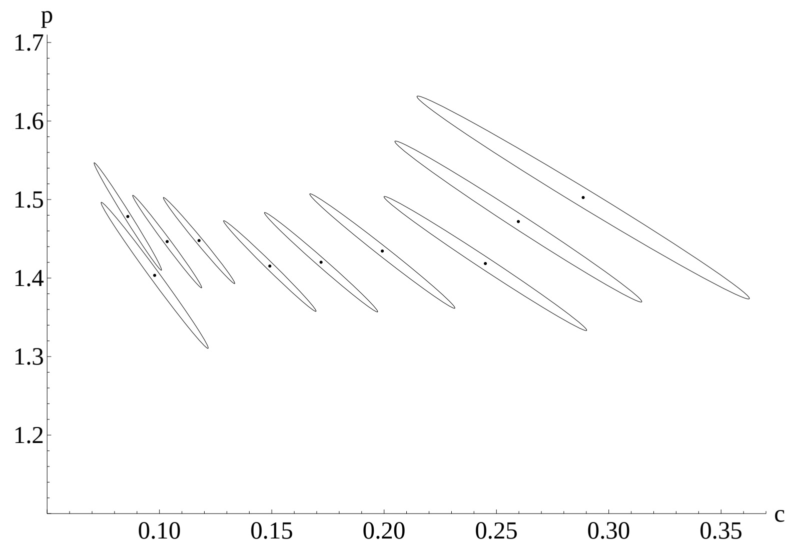


Figure 4.5: The ellipses show the 2σ allowed regions of the $p-c$ plane, cf. equation (4.1), for $\omega = 0.1, 0.2, \ldots, 1.0$, from left to right, with $2 \leq N \leq 23$. As ω increases (so that for fixed F the cosmological constant decreases), c increases substantially, while p increases slightly.

ied in $[0.1, 1]$. There is a distinct increase in c as ω increases, while p shows a barely significant increase, so that overall the probability of positivity drops substantially as ω increases. This trend can be understood as follows: increasing ω at fixed F reduces $\mathcal{H}_{\text{shift}}$, and hence shifts the entire spectrum toward more negative values, making a fluctuation to positivity more improbable.

We note that for fixed F, increasing ω *reduces* the cosmological constant, so within this class of critical points, the probability of stability is higher at higher cosmological constant. One should not read too much into this, however, as one can increase the cosmological constant by increasing $F, |W|, m_{\text{susy}}$ by a common factor without affecting the probability of stability.

4.5 Stability of Critical Points with Small F-terms

The conclusions of §4.4 apply to a generic critical point, by which we mean one at which the functions K, W are random functions that do not automatically manifest any special hierarchies.[7] However, it is far from clear a priori that a typical metastable vacuum arises from among the set of generic critical points: a tiny subclass of critical points that enjoy a high likelihood of stability as a consequence of some special structure might well account for most of the metastable vacua.

As originally noted by Denef and Douglas in [33], a particularly interesting class of critical points are those at which the F-terms are small compared to the supersymmetric masses: approximate supersymmetry can be expected to make stability more likely. Specifically, we will consider de Sitter critical points at which

$$\sqrt{3}|W| < F \ll |Z_{ab}| \sim |U_{abc}| \,. \tag{4.55}$$

In this section we will reexamine the stability of critical points in this approximately-supersymmetric corner of the supergravity landscape. As will become clear, our conclusion differs from that of Denef and Douglas, and we will carefully explain the reason for the disparity.

We will see that eigenvalue repulsion in the mass matrix between the bulk of the eigenvalues and the Goldstino direction typically generates at least two tachyonic directions, rendering generic critical points unstable. This effect significantly influences the fine-tuning needed to obtain a metastable de Sitter solution in supergravity. Through numerical simulations and through statistical

[7]Of course, large ratios can arise in this setting by chance, but this possibility is already encoded in the results of §4.4.

analysis we will show that metastable critical points constitute an exponentially small fraction of all critical points: $N_{\text{vacua}} \simeq N_{\text{crit.pts.}} e^{-cN^p}$, with $p \approx 1.3$ and $c \approx 0.08$.

4.5.1 The Denef-Douglas landscape of de Sitter vacua

In order to analyze the stability properties of the mass matrix in the regime where $F \ll m_{\text{susy}}$, we write $\mathcal{H}$ as [33],

$$\mathcal{H} = V_0'' + V_1'' + V_2'' , \tag{4.56}$$

with

$$V_0'' = (\mathcal{M} + |W|\mathbb{1})(\mathcal{M} - 2|W|\mathbb{1}) , \tag{4.57}$$

$$V_1'' = \begin{pmatrix} 0 & S_1 \\ \bar{S}_1 & 0 \end{pmatrix} , \quad S_1 = U_{abc}\bar{F}^c , \tag{4.58}$$

and

$$V_2'' = \begin{pmatrix} S_2 & 0 \\ 0 & \bar{S}_2 \end{pmatrix} , \quad S_2 = \delta_{a\bar{b}}F^2 - F_a\bar{F}_{\bar{b}} - R_{a\bar{b}c\bar{d}}\bar{F}^c F^{\bar{d}} , \tag{4.59}$$

where $\mathcal{M}$ is the matrix given in equation (4.6).

When $m_{\text{susy}} \gg F$, all but two of the eigenvalues of $\mathcal{H}$ are generically of order m^2_{susy}, and are predominantly determined by V_0'', with corrections of order F from V_1'', and of order F^2 from V_2''. However, the critical point condition, equation (4.6), requires that $\mathcal{M}$ has an eigenvalue $2|W|$, with the corresponding eigenvector pointing in the Goldstino direction. The eigenvalues of $\mathcal{M}$ come in pairs differing only in sign, and the eigenvalues $\lambda_\pm = \pm 2|W|$ of $\mathcal{M}$ correspond to eigenvalues $m_0^2 = 0$, $m_0^2 = 4|W|^2$ of V_0''. The larger of these 'Goldstino' eigenvalues is $O(F^2)$, so that one cannot a priori neglect the effects of V_1'', V_2'' on the stability of the Goldstino direction. This section is dedicated to a careful examination of these effects.

Setting $F_a = \delta_a{}^1 F e^{i\vartheta_F}$ and performing a unitary transformation that diagonalizes $Z_a{}^{\bar{c}} \bar{Z}_{\bar{b}\bar{c}}$, we obtain the simplified mass matrix

$$\mathcal{H}_\star = \left(\begin{array}{c|c} m^2_{a\bar{b}} & m^2_{ab} \\ \hline m^2_{\bar{a}\bar{b}} & m^2_{\bar{a}b} \end{array}\right) = \left(\begin{array}{cc|cc} m^2_{1\bar{1}} & \mathcal{O}(F^2) & \multicolumn{2}{c}{\multirow{2}{*}{m^2_{ab}}} \\ \mathcal{O}(F^2) & \mathrm{diag}(\lambda^2_{a'}) & & \\ \hline \multicolumn{2}{c|}{\multirow{2}{*}{$m^2_{\bar{a}\bar{b}}$}} & m^2_{1\bar{1}} & \mathcal{O}(F^2) \\ & & \mathcal{O}(F^2) & \mathrm{diag}(\lambda^2_{a'}) \end{array}\right). \tag{4.60}$$

The eigenvalues of $Z_a{}^{\bar{c}} \bar{Z}_{\bar{b}\bar{c}}$ have been denoted $\lambda^2_{a'}$, for $a' = 2, \ldots N$, while by the critical point equation we have $m^2_{1\bar{1}} = 2|W|^2 - R_{1\bar{1}1\bar{1}} F^2$. In the approximately-supersymmetric regime, $\lambda^2_{a'} \gg m_{\mathrm{susy}} F$, and we have correspondingly omitted $\mathcal{O}(F^2)$ contributions to the diagonal entries $(\mathcal{H}_\star)_{a'\bar{a}'}$ for $a' = 2, \ldots N$. The notation $\mathcal{H}_\star$ emphasizes that the matrix appearing in (4.60) is not a truncation of $\mathcal{H}$ to some order in F. Instead, $\mathcal{H}_\star$ has been strategically simplified so that, while it efficiently yields results for the two smallest eigenvalues of $\mathcal{H}$ that are accurate up to $\mathcal{O}(F^3)$ corrections, the higher eigenvalues of $\mathcal{H}_\star$, which are generically positive in any case, do not coincide with those of $\mathcal{H}$ to this accuracy.

Following the discussion of [33], we focus on the submatrix spanned by the normalized eigenvectors of $\mathcal{M}$ with eigenvalues $\pm 2|W|$, which in the above basis can be expressed as

$$(\Psi^+_{11})_a = \frac{1}{\sqrt{2}}(e^{i\Delta\vartheta}\,\delta_a{}^1 + e^{-i\Delta\vartheta}\delta_a{}^{N+1}) \;\text{ and }\; (\Psi^-_{11})_a = \frac{i}{\sqrt{2}}(e^{i\Delta\vartheta}\,\delta_a{}^1 - e^{-i\Delta\vartheta}\,\delta_a{}^{N+1})\,, \tag{4.61}$$

where $\Delta\vartheta = \vartheta_F - \vartheta_W$. Neglecting for the moment mixings with the other eigenvectors of $\mathcal{M}$, this 2×2 Goldstino submatrix of the full mass matrix is

$$\mathcal{H}_{\text{sub}} = \begin{pmatrix} m^2_{1\bar{1}} & m^2_{11} \\ m^2_{\bar{1}\bar{1}} & m^2_{1\bar{1}} \end{pmatrix}. \tag{4.62}$$

While a diagonalization of this subsystem by itself does *not* in general correspond to a diagonalization of the corresponding directions in the full mass matrix, it is instructive to attempt to treat the off-diagonal mixings in perturbation theory. The eigenvalues of the submatrix are given by

$$h^{\pm} = m^2_{1\bar{1}} \pm |m^2_{11}|\,, \tag{4.63}$$

which can be written as

$$h^{\pm} = 2|W|^2 - R_{1\bar{1}1\bar{1}}F^2 \pm \left| U_{111} F e^{-\vartheta_F} - 2|W|^2 e^{2i(\vartheta_F - \vartheta_W)} \right|. \tag{4.64}$$

The dominant contribution to $h^{\pm}$ for $|U_{111}| \sim |R_{1\bar{1}1\bar{1}}| \sim \mathcal{O}(F^0)$ is the term $U_{111}F$, so that generically $h^- < 0$.

In [33], it was observed that fine-tuning $|U_{111}|$ to be $\mathcal{O}(F)$ is necessary for stability of the mass matrix. However, [33] also argued that this condition is sufficient, and concluded that metastable critical points are fairly common in a supergravity landscape. We will now show that the eigenvalues $h^{\pm}$ of the submatrix $\mathcal{H}_{\text{sub}}$ of $\mathcal{H}$ cannot be regarded as good approximations to the actual eigenvalues of $\mathcal{H}$. Upon computing the leading-order corrections to (4.63), we will find that metastable critical points constitute an exponentially small fraction of all de Sitter critical points.

4.5.2 Eigenvalue repulsion induces tachyons

In this section we discuss the correction to the eigenvalues of the mass matrix induced by V_1'', equation (4.58). To determine the two smallest eigenvalues to

$\mathcal{O}(F^2)$, we may neglect $\mathcal{O}(F^2)$ contributions to $m^2_{a'\bar{b}'}$ for $a' \neq b'$. With this simplification, equation (4.60) can be written as

$$\mathcal{H}_\star \simeq \left(\begin{array}{cc|cc} m^2_{1\bar{1}} & 0 & s & v^T_{a'} \\ 0 & \mathrm{diag}(\lambda^2_{a'}) & v_{a'} & T_{a'b'} \\ \hline s^* & v^\dagger_{\bar{a}'} & m^2_{1\bar{1}} & 0 \\ v^*_{\bar{a}'} & T^*_{\bar{a}'\bar{b}'} & 0 & \mathrm{diag}(\lambda^2_{a'}) \end{array}\right). \tag{4.65}$$

In equation (4.65) we have introduced the $U(N-1)$ scalar $s = m^2_{11}$, the vector $v_{a'} = m^2_{1a'}$, and the symmetric tensor $T_{a'b'} = m^2_{a'b'}$. The unitary transformation

$$U = \left(\begin{array}{cc|cc} -\frac{e^{i\alpha}}{\sqrt{2}} & 0 & \frac{e^{i\alpha}}{\sqrt{2}} & 0 \\ 0 & \delta_{a'\bar{b}'} & 0 & 0 \\ \hline \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \delta_{a'\bar{b}'} \end{array}\right), \tag{4.66}$$

with $\alpha = \arg(s)$, diagonalizes the sub-matrix $\mathcal{H}_{\rm sub}$, i.e.

$$\widetilde{\mathcal{H}}_\star = U^\dagger \mathcal{H}_\star U = \left(\begin{array}{cc|cc} h^- & u^\dagger_{a'} & 0 & w^T_{a'} \\ u_{a'} & \mathrm{diag}(\lambda^2_{a'}) & u_{a'} & T_{a'b'} \\ \hline 0 & u^\dagger_{\bar{a}'} & h^+ & -w_{a'} \\ w^*_{a'} & T^*_{\bar{a}'\bar{b}'} & -w^*_{a'} & \mathrm{diag}(\lambda^2_{a'}) \end{array}\right), \tag{4.67}$$

where $u_{a'} = \frac{1}{\sqrt{2}} v_{a'}$, $w_{a'} = -\frac{1}{\sqrt{2}} e^{-i\alpha} v_{a'}$, and $h^\pm$ is given by equation (4.63). The conclusion of [33] is that modest fine-tuning of s ensures the positivity of $h^\pm$, and hence of $\mathcal{H}$. Here we investigate the effect of the vector $v_{a'}$ on the eigenvalues of $\mathcal{H}$. The leading-order effect of the tensor $T_{a'b'}$ is to induce $\mathcal{O}(m_{\rm susy}F)$ shifts of the eigenvalues $\lambda^2_{a'} \gg m_{\rm susy}F$, so that we may consistently neglect $T_{a'b'}$. The characteristic polynomial of $\widetilde{\mathcal{H}}_\star$ is then given by

$$\begin{aligned} C(\rho) &= C_0(\rho)\Bigg[1 - \sum_{b'=2}^{N} \frac{|v_{b'}|^2}{(h^+ - \rho)(\lambda^2_{b'} - \rho)} - \sum_{b'=2}^{N} \frac{|v_{b'}|^2}{(h^- - \rho)(\lambda^2_{b'} - \rho)} \\ &\quad + \sum_{a',b'=2}^{N} \frac{|v_{a'}|^2 |v_{b'}|^2}{(h^+ - \rho)(h^- - \rho)(\lambda^2_{a'} - \rho)(\lambda^2_{b'} - \rho)}\Bigg], \end{aligned} \tag{4.68}$$

where $C_0(\rho)$ denotes the characteristic polynomial for $v_{a'} = 0$, i.e.

$$C_0(\rho) = (h^- - \rho)(h^+ - \rho) \prod_{a'=2}^{N} \left(\lambda_{a'}^2 - \rho\right)^2 . \tag{4.69}$$

The leading-order effect of the vector $v_{a'}$ is evidently to induce an 'interaction' between the approximate eigenvalues $h^\pm$ and $\lambda_{a'}^2$. This interaction is a manifestation of eigenvalue repulsion, and indeed, the effect of each term in the sum is to increase the splitting between $h^\pm$ and $\lambda_{a'}^2$. Restricting the polynomial to small values of ρ close to the smallest eigenvalues of the mass matrix and dividing by the overall factors of the larger eigenvalues, equation (4.68) can be rewritten as

$$\begin{aligned} \frac{C(\rho)}{\prod_{a'=2}^{N} \lambda_{a'}^2} &= (h^+ - \rho)(h^- - \rho) - (h^- - \rho) \sum_{b'=2}^{N} \frac{|v_{b'}|^2}{\lambda_{b'}^2} - (h^+ - \rho) \sum_{b'=2}^{N} \frac{|v_{b'}|^2}{\lambda_{b'}^2} \\ &+ \sum_{a',\, b'=2}^{N} \frac{|v_{a'}|^2\, |v_{b'}|^2}{\lambda_{a'}^2 \lambda_{b'}^2} . \end{aligned} \tag{4.70}$$

The smallest eigenvalues of the mass matrix are thus given by

$$\begin{aligned} m_\pm^2 &= h^\pm - \sum_{b'=2}^{N} \frac{|v_{b'}|^2}{\lambda_{b'}^2} = m_{1\bar{1}}^2 \pm |m_{11}^2| - \sum_{b'=2}^{N} \frac{|m_{1b'}^2|^2}{\lambda_{b'}^2} \\ &= 2|W|^2 + K_{11}{}^{e} K_{\bar{1}\bar{1}e} F^2 - K_{1\bar{1}1\bar{1}} F^2 \pm \left| U_{111} F e^{-\vartheta_F} - 2|W|^2 e^{2i(\vartheta_F - \vartheta_W)} \right| \\ &- F^2 \sum_{b'=2}^{N} \frac{|U_{11b'}|^2}{\lambda_{b'}^2} . \end{aligned} \tag{4.71}$$

This is one of our main results. The smallest eigenvalue m_-^2 of the Hessian matrix $\mathcal{H}$ differs from that of [33] by the non-positive term $-F^2 \sum_{b'=2}^{N} \frac{|U_{11b'}|^2}{\lambda_{b'}^2}$, in a manifestation of eigenvalue repulsion between the Goldstino and the supersymmetrically stabilized moduli with masses of order $\lambda_{b'}^2$.

We now turn to assessing the impact of this contribution to m_-^2.

4.5.3 Eigenvalue fluctuations and de Sitter vacua

In this section we will determine the probability that a randomly chosen approximately-supersymmetric critical point is metastable by computing the probability that m_-^2, as given in equation (4.71), is positive. Our approach is to determine the statistical properties[8] of each term in the sum, i.e. we will obtain the cumulative distribution function (cdf) for each term, from which the corresponding probability density function (pdf) can be obtained by differentiation. Although in principle one might hope to convolve the constituent probability density functions to obtain the pdf of m_-^2, this is rather involved. Fortunately, we will find that one of the terms of equation (4.71) dominates both in magnitude and in the probability of fluctuations, and it suffices to examine this term.

For the analytical estimates provided here, we will assume $\Omega = \mathcal{N}(0, \frac{1}{\sqrt{N}})$, though similar arguments could be made for e.g. the uniform distribution.

We find it convenient to rewrite (4.71) as

$$m_-^2 = F^2\mathcal{T} + F^2\mathcal{S} \tag{4.72}$$

with

$$\mathcal{T} = \frac{2}{3}\omega^2 + K_{11}{}^{e}K_{\bar{1}\bar{1}e} - K_{1\bar{1}1\bar{1}} - |t_{\mathrm{hol}}|\,, \tag{4.73}$$

and

$$\mathcal{S} \equiv -\sum_{b'=2}^{N}\frac{|U_{11b'}|^2}{\lambda_{b'}^2}\,, \tag{4.74}$$

where

$$|t_{\mathrm{hol}}| \equiv \left|U_{111}e^{2i\vartheta_W - 3i\vartheta_F}F^{-1} - \frac{2}{3}\omega^2\right|, \tag{4.75}$$

and we have used the definition (4.31).

[8] Recall from [33] that a fine-tuning of $|U_{111}F| \lesssim F^2$ is necessary for stability, and granting this fine-tuning, all the terms in equation (4.71) are of the same order, $O(F^2)$.

Subdominant contributions

We will begin by studying the terms collected in $\mathcal{T}$, which do not involve the vector $v_{a'}$.

At the critical points of interest, $\sqrt{3}|W| \leq F$, so that the total energy density is nonnegative. Thus, the first term in equation (4.73) gives a contribution in the range $[0, \frac{2}{3}]$.

The second term of equation (4.73) is $|K^{(3)}|^2 \equiv K_{11}{}^{e} K_{\bar{1}\bar{1}e}$, which is the sum of squares of N random variables, each drawn from $\mathcal{N}(0, \frac{1}{\sqrt{N}})$. Thus, $|K^{(3)}|^2$ is distributed as $\frac{1}{N}\chi^2_N$, where χ^2_N is a chi-square distribution with N degrees of freedom. Since χ^2_N has mean N, we conclude that

$$\langle\, |K^{(3)}|^2 \,\rangle = 1\,. \tag{4.76}$$

To find the probability of fluctuations, we note that the corresponding cdf is given by

$$P\Big(|K^{(3)}|^2 \leq x\Big) = P\Big(\frac{1}{N}\chi^2_N \leq x\Big) = \frac{1}{\Gamma(N/2)}\gamma\Big(\frac{N}{2}, \frac{Nx}{2}\Big)\,, \tag{4.77}$$

where γ denotes the lower incomplete gamma function. The asymptotic behavior can be obtained as follows: by the central limit theorem, the cdf of a chi-square distributed variable for $N \gg 1$ degrees of freedom tends to that of a Gaussian distributed variable with unit variance, $P(\chi^2_N \leq y) \approx P(\mathcal{N}(0,1) \leq x)$, where $x = \frac{y-N}{\sqrt{2N}}$ [170].

We are particularly interested in the probability of $|K^{(3)}|^2$ fluctuating to a large value and thereby stabilizing the smallest eigenvalue m_-^2 of $\mathcal{H}$. As we will describe below, large in this context means $O(N)$, so that we consider

$$P\Big(|K^{(3)}|^2 \leq N\Big) \approx P\Big(\mathcal{N}(0,1) \leq 2^{-1/2}N^{3/2}\Big)\,, \tag{4.78}$$

for $N \gg 1$, from which we obtain

$$P\Big(|K^{(3)}|^2 \geq N\Big) \lesssim \frac{1}{\sqrt{\pi}N^{3/2}}\, e^{-\frac{N^3}{4}} \,. \tag{4.79}$$

Fluctuations of $|K^{(3)}|^2$ may therefore be neglected in comparison to the much more probable fluctuations we will discuss in §4.5.3.

The third term in (4.73), $K^{(4)}_{1\bar{1}1\bar{1}}$, is normally distributed with a vanishing expectation value, and with a variance no larger than $\frac{2}{N}$, so that large deviations of the order N are likewise so improbable as to be negligible:

$$P\Big(K^{(4)}_{1\bar{1}1\bar{1}} \geq N\Big) \sim e^{-N^3} \,. \tag{4.80}$$

The fourth term in (4.73), $-|t_{\rm hol}|$, is negative semidefinite, and only one *entry* (not eigenvalue) of $\mathcal{H}$, namely U_{111}, needs to be adjusted in order to change the size of $|t_{\rm hol}|$. Therefore, it is straightforward to fine-tune $|t_{\rm hol}|$ to be small. It is clear from the discussion above that, as originally noted in [33], m_-^2 is generically negative unless U_{111} is fine-tuned to make $|t_{\rm hol}| \lesssim O(1)$. For our goal of obtaining a conservative estimate of the probability that $m_-^2 > 0$, it suffices to set $|t_{\rm hol}| = 0$.

The eigenvalue repulsion term

Finally, the last term in equation (4.72) is the sum of squares of $N-1$ terms. The numerators of the terms in equation (4.74) are the squares of independent normally distributed variables, while the denominators are the squares of the eigenvalues of $\mathcal{M}$.

The eigenvalues of $\mathcal{M}$ range from around $O(\frac{1}{N})$ to 2 in units[9] of $m_{\rm susy}$, so

[9]Since by assumption $U_{abc} \sim Z_{ab}$, the dependence on the supersymmetric mass scale $m_{\rm susy}$ cancels between the numerator and denominator in equation (4.74).

$\langle|\mathcal{S}|\rangle \sim \mathcal{O}(N)$. Recalling that the contributions to $\mathcal{T}$ have mean sizes independent of N, we conclude that $\mathcal{S}$ provides the dominant contribution to m_{-}^{2} at large N. Since $\langle\mathcal{S}\rangle < 0$, this term *destabilizes generic critical points in the approximately-supersymmetric regime.*

To determine the (small) probability that m_{-}^{2} is nevertheless positive, we will now estimate the probability that $\mathcal{S} \gtrsim -1$, so that $\mathcal{T} + \mathcal{S}$ can be positive. First, we recognize that in light of the discussion in §4.3, fluctuations that increase the denominators appearing in $\mathcal{S}$, corresponding to inward fluctuations of the eigenvalues of a Wishart matrix, are extremely unlikely at large N [167]. Fluctuations of $\mathcal{S}$ toward smaller magnitude are principally determined by the fluctuations of the numerators. (We have explicitly verified this in simulations.) This justifies simplifying the problem by fixing the factors of $\lambda_{b'}^{2}$ to their mean values, $\langle\lambda_{b'}^{2}\rangle$, as determined by the bulk distribution given by equation (4.38). Henceforth we consider the sum

$$\mathcal{S}' = \sum_{b'=2}^{N} \frac{|U_{11b'}|^{2}}{\langle\lambda_{b'}^{2}\rangle}\,, \tag{4.81}$$

which is the weighted sum of $N-1$ variables that are all independently distributed as χ_{1}^{2}. Weighted sums of χ^{2}-distributed variables (or, equivalently, sums of Γ-distributed variables with different scale parameters) occur frequently in statistics, and in particular in the theory of the distributions of quadratic forms. While we have not found a closed-form expression for the convolution of $N-1$ such terms, approximations for expressions like (4.81) have been developed. An approximation by Solomon and Stephens has been argued to be particularly accurate in the small-argument regime of interest [171], but we will find that for our purposes it does not constitute a close approximation to the cumulative probability for small arguments. By matching the first three algebraic moments μ_1, μ_2, μ_3 of $\mathcal{S}'$ to those of $a \cdot w^{b}$, where w is χ_{r}^{2}-distributed and a, b, and r are constants, this approximation is obtained[10] by numerically solving the equations

$$\begin{aligned} \mu_1 &= a\, 2^b \, \frac{\Gamma(b+\frac{r}{2})}{\Gamma(\frac{r}{2})}\,, && (4.82)\\ \frac{\mu_2'}{\mu_1^2} &= \Gamma\Big(\frac{r}{2}\Big)\, \frac{\Gamma(2b+\frac{r}{2})}{\Big[\Gamma(b+\frac{r}{2})\Big]^2}\,, && (4.83)\\ \frac{\mu_3'}{\mu_1^3} &= \Big[\Gamma\Big(\frac{r}{2}\Big)\Big]^2\, \frac{\Gamma(3b+\frac{r}{2})}{\Big[\Gamma(b+\frac{r}{2})\Big]^3}\,. && (4.84) \end{aligned}$$

Thus, in this approximation,

$$P\Big(\mathcal{S}' \le s\Big) \approx P\Big(a(\chi_r^2)^b \le s\Big) = P\Big(\chi_r^2 \le \Big(\frac{s}{a}\Big)^{1/b}\Big)\,. \tag{4.85}$$

As $\mathcal{T} \sim O(1)$, m_-^2 could be positive if $\mathcal{S}'$ fluctuates down to be $O(1)$, for which we obtain

$$P\Big(m_-^2 > 0\Big) \approx P\Big(\mathcal{S}' \lesssim 1\Big) \approx e^{-c\cdot N^p}\,, \tag{4.86}$$

where $c \simeq 23$, and $p \simeq 0.24$. As we will see in §4.5.4, even though this approximation qualitatively matches the shape of $\mathcal{S}'$, for $N \gg 1$ it severely overestimates the probability of a fluctuation of $\mathcal{S}'$ to be of $O(1)$, and it remains an open question to obtain a good analytic or semi-analytic approximation of equation (4.81).

4.5.4 Numerical results

Figure 4.6 shows a histogram of m_-^2 and its constituent terms $\mathcal{T}$ and $\mathcal{S}$, for $N = 40$. It is clear that $\mathcal{S}$ gives the dominant contribution to m_-^2. Moreover, the narrow support of the $\mathcal{T}$ histogram illustrates the finding of §4.5.3 that large fluctuations of $\mathcal{T}$ are much less probable than correspondingly large fluctuations of $\mathcal{S}$.

[10]There is a misprint in the fifth equation of §3.1 of [171].

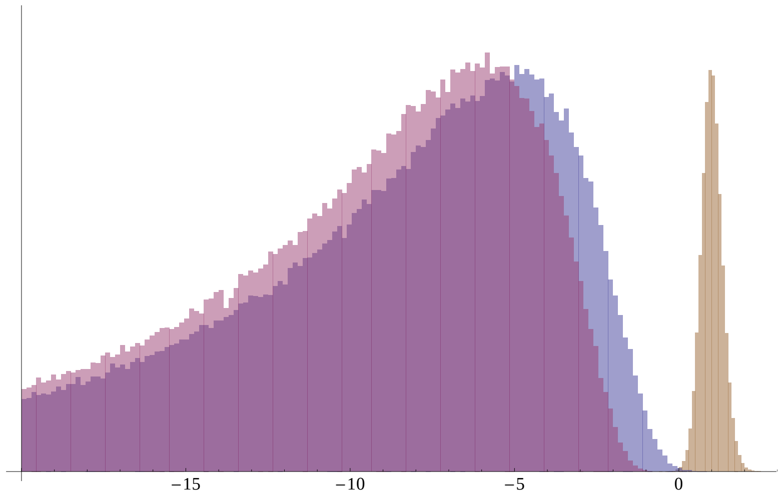


Figure 4.6: Histograms of the smallest eigenvalue m_-^2, cf. equation (4.72), and its constituent terms $\mathcal{T}$ and $\mathcal{S}$, for $N = 40$, in units of F^2. The eigenvalue repulsion sum $\mathcal{S}$ has the leftmost peak, the total mass m_-^2 has the central peak, and $\mathcal{T}$ appears on the right. Note that $\mathcal{S}$, and consequently m_-^2, has support over a range of size N (not fully shown in the figure), while $\mathcal{T}$ has variance $\frac{2}{N}$.

Finally, Figure 4.7 presents the result of simulations of the mass matrix in the approximately-supersymmetric regime. (The value of ω has a negligible effect on stability in this regime.) The data agrees well with (4.1), with $p = 1.28 \pm 0.03$ and $c = 0.083 \pm 0.008$.[11] This is a much larger value for p than that obtained by the analytical estimate of §4.5.3, so that the latter gives an extremely conservative upper bound on the asymptotic large N probability of positivity.

4.6 Beyond Random Supergravity

In this section we will discuss potential extensions of our assumptions (§4.6.1), explain the consequences of decoupling for the probability of positivity (§4.6.2), and illustrate our results in the example of the KKLT scenario (§4.6.3).

[11] To obtain a conservative bound, we fit to the data points with $N \geq 7$.

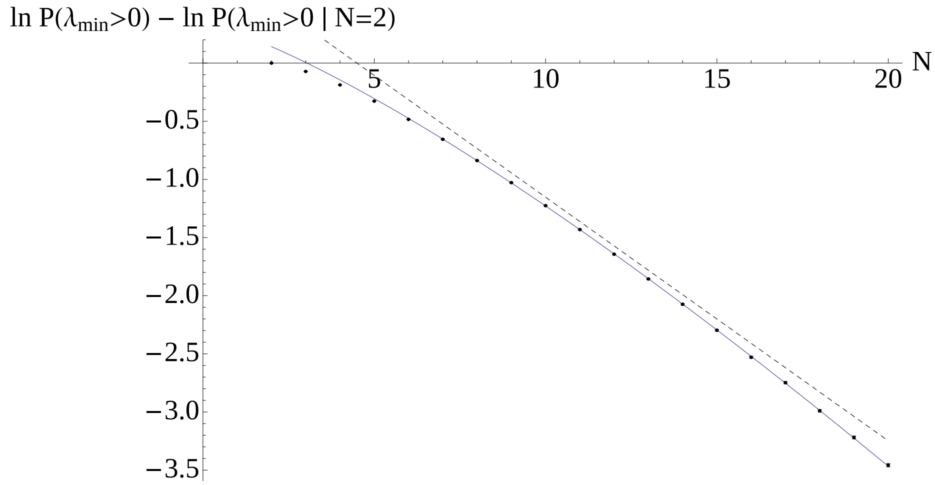


Figure 4.7: The logarithm of the probability $P(\lambda_{\min} > 0)$ that the smallest eigenvalue of $\mathcal{H}$ is positive, as a function of N. Each point corresponds to 10^6 realizations of the full mass matrix, and the error bars give the 2σ statistical uncertainty. The curve shows the best fit to equation (4.1), with $p = 1.28 \pm 0.03$ and $c = 0.083 \pm 0.008$. The dashed line with $p = 1$ is for reference.

4.6.1 Universality

The results of §4.4 and §4.5 were obtained from the assumptions enumerated in §4.2.3: most notably, K and W were taken to be random functions of N scalar fields, so that their various (appropriately covariant) derivatives are i.i.d. variables drawn from a distribution $\Omega(\mu, \sigma)$. Equivalent assumptions are standard in the statistical study of flux compactifications, cf. [142], and in particular are fully consistent with the assumptions of [33]. Nevertheless, in this section we will venture a few remarks about possible extensions of this simplest definition of a random supergravity.

First, we have taken the random variables to be normally distributed, $\Omega = \mathcal{N}(0, \frac{1}{\sqrt{N}})$, throughout this work, and now we justify this assumption. The celebrated phenomenon of universality in random matrix theory ensures that at large N the eigenvalue spectrum, and also the fluctuations of extreme eigenvalues, are independent of the statistical details of the inputs. Universality has been demonstrated in a staggering array of physical and mathematical systems, many with $N \lesssim \mathcal{O}(10^2)$, including interfaces in liquid crystals [172], the timing of buses in Cuernavaca [173], and the power output of coupled lasers [174]. See [175, 176] for overviews of universality and [177, 178] for results with close connections to ensembles studied here; extensions to ensembles in which the matrix entries have power-law tails include [179]. The lesson is that the particular choice of Ω is immaterial, provided that the moments of Ω are appropriately bounded. (One should compare distributions Ω_1, Ω_2 yielding the same root-mean-square size for entries in $\mathcal{H}$, as this sets the physical scale.)

Despite the strong expectation that universality should be applicable for our system, it is still reasonable to ask whether the values of N in our analysis are large enough for these asymptotic results to apply in practice. We have addressed this point directly by repeating our simulations for different choices of distribution, with excellent agreement.

A more fundamental question is whether in the effective theories derived from string compactifications, the derivatives of K and W are accurately modeled as i.i.d. variables drawn from *any* distribution, or if instead these quantities are not i.i.d.[12] Microphysical constraints, for example the relics of extended supersymmetry,[13] might be expected to introduce correlations among these variables, as in the special geometry relation of (4.24), cf. [33, 145], so that the derivatives of K and W are not all independent. A definitive answer to this question is beyond the scope of this work, but it is encouraging that in simpler cases such as the Wigner ensemble, universality has been shown to apply to matrices with

[12]We thank M. Douglas for instructive correspondence on this point.

[13]See [180] for related work in maximal supergravity.

highly correlated entries [181].

It would also be interesting to understand the possible impact of global constraints on our considerations. We have studied the ensemble of critical points arising in a general supergravity theory, and we have taken the derivatives of K and W evaluated at each such point to be random functions. To understand the distribution of vacua within the moduli space, one should incorporate further structure. The (index) density of supersymmetric vacua is well known to be correlated with the curvature of the moduli space [30], while the global structure of the superpotential is better modeled as a random holomorphic section of a line bundle over the moduli space (see [182] for a definitive treatment of the density of supersymmetric vacua in this context). Extending the study of non-supersymmetric vacua to this level of detail is an interesting problem for the future.

One might expect that constraints from Morse theory will require some small deviations from the purely statistical results obtained here.[14] The random matrix ensembles we have described (as in [33] and earlier works) predict certain ratios between the numbers of saddle points of varying index, which are not automatically consistent with the Morse inequalities. The necessary adjustments can be accommodated without changing the number of minima relative to saddle points, and we find it plausible that any effect on the relative number of minima can be neglected.

[14] We thank B. Czech for very helpful correspondence about these constraints.

4.6.2 Decoupling improves stability

A significant assumption in our analysis is that W and K are general random functions of N scalars. In physically well-motivated examples, there can of course be two or more sectors of fields with distinct mass scales. For instance, consider[15] a two-sector supergravity theory with $N = N_H + N_L$ fields, in which the heavy scalars ϕ_H^a, $a = 1, \ldots N_H$, receive large supersymmetric masses m_H, and supersymmetry is dynamically broken in a decoupled system of lighter scalars ϕ_L^i, $i = 1, \ldots N_L$, at a much lower scale m_L.

Explicitly, such a model can be constructed from a superpotential and a Kähler potential that are additively separable. In a convenient Kähler gauge, one has

$$\begin{aligned} K(\phi_H, \bar{\phi}_H, \phi_L, \bar{\phi}_L) &= K_H(\phi_H, \bar{\phi}_H) + K_L(\phi_L, \bar{\phi}_L)\,, \\ W(\phi_H, \phi_L) &= W_H(\phi_H) + W_L(\phi_L)\,. \end{aligned} \tag{4.87}$$

By assumption $Z_{ab} \sim m_H$ and $Z_{ij} \sim m_L$, while by (4.87), the cross-couplings in the supersymmetric mass matrix are small: $Z_{ai} = K_a W_i \sim \mathcal{O}(F)$. Thus, at small F, $\mathcal{H}_{\rm susy}$ separates into two distinct Wishart matrices. (If F is not small compared to m_L, or if the separability of the superpotential is imperfect, then the off-diagonal masses in $\mathcal{H}_{\rm susy}$ cannot be neglected.)

A cautionary remark is necessary at this point. The masses-squared in a supersymmetrically-stabilized sector are not necessarily positive: setting $F = 0$ in equation (4.13), the contribution of $\mathcal{H}_{\rm susy}$ is nonnegative, but $\mathcal{H}_{\rm shift}$ and $\mathcal{H}_{\rm pure}$ make tachyonic contributions that will be significant unless $W \ll m_{\rm susy}$. Of course, the resulting masses do obey the Breitenlohner-Freedman (BF) bound,

[15] We are indebted to S. Kachru for emphasizing the importance of this example.

but this in itself does not guarantee that this sector will remain stable after supersymmetry is broken badly enough to make the cosmological constant positive.

We now recall from §4.5 that for $F \ll m_{\text{susy}}$, superpotential couplings of the form $\frac{|U_{11A'}|^2}{\lambda_{A'}^2}$ contribute to the destabilization of the Goldstino direction, cf. equation (4.71), where A' runs over all fields. The numerator of this contribution to the Goldstino mass from the heavy, supersymmetric subsystem is

$$|U_{11a}|^2 = |\mathcal{D}_a Z_{11}|^2 = |\partial_a Z_{11} + K_a Z_{11}|^2 \,, \tag{4.88}$$

which under the decoupling assumptions of equation (4.87) is of order $|W|^2$. (For a non-decoupled system one finds instead $|U_{11a}|^2 \sim m_{\text{susy}}^2$.)

Since the denominator λ_a^2 is of order m_H^2, the separability of equations (4.87) leads to a suppression of order $\frac{|W|^2}{m_H^2}$ of the heavy fields' negative contribution to the Goldstino direction mass-squared. Thus, even for a modest hierarchy between the supersymmetric masses, $m_H^2 \gtrsim N_H |W|^2$, the high-scale sector decouples, and does not contribute significantly to the mass of the Goldstino.

In conclusion, the relevant number of fields for the stability analysis of §4.5 is N_L, the number of 'light' fields that participate in dynamical supersymmetry breaking (the superpartner of the Goldstino is assumed to be entirely among these fields.) The fraction of critical points that are metastable is then proportional to $\exp(-cN_L{}^p)$. Provided that the heavy sector, taken in isolation, contains a number of supersymmetric vacua that is exponential in N_H, then the net result, for $N_H \gg N_L$, is a mild reduction in the number of metastable vacua.

4.6.3 Stability in the KKLT scenario

The KKLT scenario [26] provides a useful setting to illustrate our findings. Consider a model with $N_K \equiv h^{1,1}_{(+)}$ Kähler moduli T_i, $i = 1, \dots N_K$, and $N_C \equiv h^{2,1}$ complex structure moduli ζ_a, $a = 1, \dots N_C$. Suppose that the superpotential takes the form

$$W = \int G \wedge \Omega + \sum_{i=1}^{N_K} \mathcal{A}_i(\zeta) \exp\left(\frac{2\pi}{n_i} T_i\right) , \tag{4.89}$$

where n_i is the dual Coxeter number for superpotential terms generated by gaugino condensation, $n_i = 1$ for terms generated by Euclidean D3-branes, and $\int G \wedge \Omega$ depends on the ζ_a. Finding a compactification with many moduli for which each Kähler modulus appears in the nonperturbative superpotential is a difficult task (cf. [183, 184] for detailed examples). Our purpose is to show that, *granting* a superpotential of the form (4.89), then for $N_K \gg 1$, an exponentially small fraction of de Sitter critical points are metastable vacua.

An important scale in the problem is the flux scale $m_{\rm flux}$, which sets the typical size of the supersymmetric masses for the ζ_a. In light of the very large number of choices of quantized flux [29], one can find configurations in which the vacuum expectation value of the classical superpotential obeys $\langle \int G \wedge \Omega \rangle \ll m^3_{\rm flux}$. This fine-tuning is necessary in order to obtain a parametrically controlled vacuum with a reasonably small cosmological constant. Given such a flux superpotential, one can find [26] a supersymmetric AdS vacuum with all moduli stabilized.

Our goal is to assess the stability of such a configuration after uplifting to de Sitter space. As a conservative first step, we imagine that the uplifting increases the cosmological constant without creating new instabilities, as a (fictitious) moduli-independent D-term would do. We expect that more plausible sources of positive energy will worsen any instability problems seen in this simple case.

To begin, we will examine the masses in the supersymmetric AdS vacuum, and ask whether these masses can be made positive definite and large compared to $|W|$.[16] If they can, then after a rigid uplifting to de Sitter space (in the sense described above), the mass matrix will be dominated by $\mathcal{H}_{\rm susy}$, which is positive definite.

The dependence of $\mathcal{A}_i$ on the ζ_a can be neglected self-consistently for Z_{ab}, but since the nonperturbative contributions lead to an imperfect separability of the superpotential, mass mixings through terms of the form Z_{ai} cannot be neglected, and the scale of the entries Z_{ai} and Z_{ij} is now $|W|$. Specifically,

$$Z_{ij} \equiv \mathcal{D}_i\mathcal{D}_j W = \partial_i\partial_j W + \left(K_{ij} - K_i K_j\right) W \,, \tag{4.90}$$

$$Z_{aj} \equiv \mathcal{D}_a\mathcal{D}_j W = \partial_a\partial_j W - K_j K_a W \,, \tag{4.91}$$

where we have used the F-flatness conditions $D_i W = D_a W = 0$. The derivatives of K will not lead to enhancements in a controllable regime, while from (4.89) it follows that $\partial_i\partial_j W \sim \partial_a\partial_j W \sim W$.

The entries of $\mathcal{H}_{\rm susy}$ with both indices in the complex structure directions are of order $m^2_{\rm flux} \gg |W|^2$, while the mixed entries in $\mathcal{H}_{\rm susy}$ receive contributions of order $m_{\rm flux}\,|W|$ from terms of the form $Z_a{}^{\bar{c}}\bar{Z}_{\bar{j}\bar{c}}$. The entries in the Kähler moduli directions are of order $|W|^2$. As in the discussion in §4.6.2, the eigenvalues of $\mathcal{H}$ split into two groups: the first consisting predominantly of the complex structure moduli and axiodilaton, which are stabilized at a high scale without BF-allowed tachyons, and the second consisting predominantly of the Kähler moduli, which have masses of order $|W|$. Since the supersymmetric Kähler moduli masses are not parametrically larger than the negative shift term $\mathcal{H}_{\rm shift}$, or the

[16]We assume throughout that $F \lesssim \text{few} \times |W|$.

off-diagonal contribution $\mathcal{H}_{\text{pure}}$, BF-allowed tachyons are typically abundant in the Kähler moduli sector in the supersymmetric AdS vacuum.

Assuming a rigid uplifting to a de Sitter critical point, we recognize that the Kähler moduli sector constitutes a particular variant of the analysis of §4.4 with $F = 0$: the somewhat more favorable regime described in §4.5, which requires $|W| \ll |Z_{ij}|$, is inaccessible. Notice that in the generic regime of §4.4, the Goldstino is by no means the only tachyon, so that instabilities will arise in the Kähler moduli sector even if the Goldstino direction belongs to some other sector, e.g. a local dynamical supersymmetry breaking sector that engineers the positive vacuum energy.

We conclude that if a system described by a superpotential of the form (4.89) is rigidly uplifted to positive vacuum energy, the fraction of de Sitter critical points that are metastable vacua is proportional to $\exp(-c\, N_K^p)$, with $p > 1$. For compactifications in which $N_K \equiv h^{1,1}_{(+)}$ is not large, this is not a serious constraint, but it has significant impact for $h^{1,1}_{(+)} \gg 1$, and particularly for $h^{1,1}_{(+)} \gg h^{2,1}$.

One might object at this point that the arguments in favor of the existence of an approximately-supersymmetric regime [33], as in §4.5, should hold for general functions W, K, so why are they not applicable here? The answer is simply that a superpotential of the form (4.89), which contains one single-instanton term for each Kähler modulus, is not a sufficiently general function. An obvious extension is to consider multiple terms (i.e., a racetrack) for each of the T_i. For the purposes of this discussion, we grant any topological prerequisites for such a multiple racetrack, e.g. we suppose that the compactification admits more than one stack of D7-branes in each homology class. Then, by fine-tuning the fluxes to adjust the prefactors $\mathcal{A}_i$, cf. [113], one can plausibly arrange that the diagonal entries of Z_{ij} are large compared to $|W|$. However, there is a statistical price for this fine-tuning, of order $(|W|/m_{\rm susy})^{N_K}$. Recalling that the boundary between the regimes of §4.4 and §4.5 occurs for $|W|/m_{\rm susy} \sim 1/N_K$, this fine-tuning is of order $N_K^{-N_K}$, which can be significant.

To recap, if one assumes a rigid uplifting that changes the cosmological constant without changing the moduli mass matrix, then for a superpotential of the form (4.89), the Kähler moduli sector will have supersymmetric masses of order $|W|$, and will be governed by the instability analysis of §4.4 (with $F = 0$, $W \neq 0$), with positivity probability $P \propto \exp(-c\, N_K^p)$, with $p > 1$. By fine-tuning a superpotential involving $\mathcal{O}(N_K)$ racetracks, requiring a statistical price $\sim N_K^{-N_K}$, one can make the supersymmetric masses large enough to guarantee stability.

The situation is considerably worse if supersymmetry is spontaneously broken by an F-term in the Kähler moduli sector: again the analysis of §4.4 applies generically, but even after fine-tuning $\mathcal{O}(N_K)$ racetracks as above, the Goldstino instability will still fall in the Kähler moduli sector, so that the instability analysis of §4.5, with $p \approx 1.3$, is applicable.

In summary, instabilities appear generic in the Kähler moduli sector after uplifting, with metastable de Sitter vacua constituting a fraction $\lesssim \exp(-c\, N_K)$ of all de Sitter critical points. However, for $h^{2,1} \gg h^{1,1}_{(+)}$, the number of KKLT vacua remains astronomically large, and the overall status of the model is not altered by our findings.

4.7 Conclusions

We have considered a general four-dimensional $\mathcal{N} = 1$ supergravity theory whose superpotential and Kähler potential are random functions of $N \gg 1$ scalar fields, and asked what fraction f of de Sitter critical points, with supersymmetry spontaneously broken by an F-term, are metastable vacua rather than unstable saddle points. Our conclusion is that an exponentially small fraction of critical points are vacua: $f \propto \exp(-cN^p)$, with $p \gtrsim 1.3$, which differs significantly from earlier results implying $f \sim \frac{1}{N}$.

The character of the instabilities that arise depends on the relative sizes of the supersymmetric and supersymmetry-breaking masses. At a generic critical point, the soft masses are comparable to the supersymmetric masses, and supersymmetry provides limited protection from instabilities. We developed a random matrix model for the Hessian matrix $\mathcal{H}$ at a generic critical point and obtained an analytic formula for its eigenvalue spectrum, finding that a significant fraction of the eigenvalues of $\mathcal{H}$ are negative. Eigenvalue repulsion makes large fluctuations of the spectrum statistically costly, and by building on the theory of fluctuations of extreme eigenvalues — and through extensive simulations of the full Hessian matrix — we argued that the probability P of a large fluctuation rendering $\mathcal{H}$ positive definite is $P \propto \exp(-cN^p)$, with $p \approx 1.5$ and $c \approx 0.3$.

Eigenvalue repulsion also controls the stability properties of approximately-supersymmetric critical points, at which the F-term F is small compared to the supersymmetric mass scale $m_{\rm susy}$. In this regime, only the two eigenvalues corresponding to the Goldstino direction risk becoming tachyonic. We computed the two smallest eigenvalues to quadratic order in F/m_{susy}, and showed that mixing with the supersymmetric masses shifts these lowest eigenvalues to negative values. We then studied the probability of a fluctuation to positivity, through analysis of the corresponding univariate statistical distribution and through simulations of the full mass matrix. In the approximately-supersymmetric regime we found $P \propto \exp(-cN^p)$, with $p \approx 1.3$ and $c \approx 0.1$.

We emphasize that the assumption that W and K are random functions — and in particular that their derivatives are independent random variables drawn from some statistical distribution — is essential. There are, however, physically-motivated situations in which W and K are *not* general random functions of all of their arguments. An important example consists of two decoupled sectors: if N_H heavy scalars receive large supersymmetric masses, and supersymmetry is dynamically broken in a decoupled system of N_L lighter scalars at a much lower scale, then for a single vacuum configuration of the light fields the corresponding number of vacua of the full system can be exponential in N_H. It seems reasonable to expect decoupling of this sort, into a 'degeneracy sector' at high scales, and a dynamical supersymmetry breaking sector at low scales, in a variety of compactifications. Restricting to the N_L light fields, our analysis suggests that the fraction of critical points that are metastable is proportional to $\exp(-cN_L^p)$. For $N_H \gg N_L$, the result is a mild reduction in the number of metastable vacua.

Let us reiterate: our finding that an exponentially small fraction of critical points in a generic supergravity theory are metastable vacua in no way excludes the existence of a tremendously large landscape of vacua. There are two primary reasons, one conceptual and one quantitative. The conceptual reason is the possibility explained in §4.6.2 and reviewed above of a decoupled system (violating our assumptions on W, K) in which the vacuum degeneracy is ensured by N_H fields that receive large supersymmetric masses. The quantitative reason is that the values of c, p that we have obtained are not so large as to entirely overwhelm the vast number of critical points in flux compactifications.

The methods and results of this work could be of use in understanding the statistical properties of the moduli mass spectrum in string compactifications, and in guiding the search for de Sitter vacua. One clear implication of our findings is that a direct search for explicit de Sitter vacua in systems with $\mathcal{O}(10)$ or more fields and reasonably general W and K is likely to be frustrated by the appearance of tachyons. Correspondingly, the most promising regimes are those in which our assumptions are strongly violated, e.g. approximately-supersymmetric critical points for which the superpartner of the Goldstino enjoys special couplings to the remaining fields. Understanding the incidence of such couplings in well-motivated supergravity theories, particularly those derived from string compactifications, is an important problem for the future.

APPENDIX A

APPENDIX FOR CHAPTER 2

A.1 Smeared Sources and Warped Sequestering

In this Appendix we show that warped sequestering in the no-scale setup of [46] survives the relaxation of an assumption made for technical simplicity in [46]: smearing of the supersymmetry-breaking anti-D3-brane around the S^3 tip of the warped deformed conifold is not required for the basic conclusion to hold.

DeWolfe, Kachru and Mulligan [58] considered a D3-$\overline{\text{D3}}$ pair smeared around the S^3 tip of a Klebanov-Strassler throat, and obtained a supergravity solution at large radius in which all fields were invariant under the $SU(2)_L \times SU(2)_R$ isometry.[1] However, it is clear that a brane placed at a particular position on the S^3 will break some of the symmetries of the system, and can be expected to source modes that are not global symmetry singlets. One can ask whether such modes will, at the *nonlinear* level, induce Φ_- perturbations, and hence D3-brane soft terms, that compete with those mediated by O_8. That is, one should ask whether soft masses computed in the smeared solution of [58] are in fact the leading soft masses in a full, unsmeared solution. We will now argue that a solution describing a single anti-D3-brane placed at the tip of the deformed conifold enjoys an $SU(2)$ symmetry in $SU(2)_L \times SU(2)_R$ and that this residual symmetry forbids the $\Delta = 5/2$ operator $\mathrm{tr}(A^i B^j)$, which might otherwise be expected to induce problematic soft masses.[2]

[1] See [90, 55, 59] for further work on supergravity solutions for antibranes in the background of [70].

[2] See [60] for a setting in which nonlinear effects of this operator indeed give the dominant contribution to the D3-brane potential.

The deformed conifold can be defined as the a subset of $\mathbb{C}^4$ satisfying

$$\det W = -\frac{\epsilon^2}{2}\,, \tag{A.1}$$

with

$$W = \begin{pmatrix} -w_3 & w_2 \\ -w_1 & w_4 \end{pmatrix} = \frac{-1}{\sqrt{2}}\begin{pmatrix} z_3 + iz_4 & z_1 - iz_2 \\ z_1 + iz_2 & -z_3 + iz_4 \end{pmatrix}. \tag{A.2}$$

The radius r is given by

$$\mathrm{tr} W^\dagger W = \sum_i |z_i|^2 = r^3. \tag{A.3}$$

In the matrix representation of the coordinates of a point p, it is easy to convince oneself that $SU(2)_L \times SU(2)_R$ acts transitively on the $T^{1,1}$ base of the cone as

$$\sigma(W(p), g) \to L(g)W(r)R(g)^\dagger\,, \tag{A.4}$$

$$g \in SU(2)_L \times SU(2)_R. \tag{A.5}$$

This means that we can choose an origin, W_0, and specify all other points by $SU(2)_L \times SU(2)_R$ transformations away from this point. It is standard to choose

$$W_0 = \begin{pmatrix} \frac{\epsilon}{\sqrt{2}} & \sqrt{r^3 - \epsilon^2} \\ 0 & -\frac{\epsilon}{\sqrt{2}}\,, \end{pmatrix}, \tag{A.6}$$

and then any point on $T^{1,1}$ at this radius can be obtained through

$$W = LW_0R^\dagger. \tag{A.7}$$

The background geometry and the smeared solution are symmetric under all these rotations. The subset of these rotations that are still symmetries once an anti-D3-brane is placed at a specific point p is by definition the stabilizer $H(p)$. Since $T^{1,1}$ is a coset space of the above $SU(2)_L \times SU(2)_R$ action, the stabilizer is (cf. e.g. [91]),

$$H(p) = \begin{cases} U(1) & r^3 > \epsilon^2 \\ SU(2) & r^3 = \epsilon^2\,. \end{cases} \tag{A.8}$$

We denote the stabilizer of our chosen origin W_0 by $SU(2)_S$, and we must impose the $SU(2)_S$ symmetry on any solution corresponding to perturbing the supergravity solution by placing an anti-D3-brane at p. Clearly, for this specific W_0, the stabilizer is the subgroup of $SU(2)_L \times SU(2)_R$ that leaves σ_3 invariant.

Now, because $\det_{i,j} \mathrm{tr}(A^i B^j) = 0$, $\mathrm{tr}(A^i B^j)$ cannot be a singlet under $SU(2)_S$. Geometrically, this is the natural outcome of the fact that $A^i B^j$ can be thought of as coordinates on the *singular* conifold. Thus, the operator $\mathrm{tr}(A^i B^j)$ has an interpretation as a point on the conifold (far) away from the tip. Such a point has a $U(1)$ stabilizer, and thus cannot be invariant under a full $SU(2)$ in $SU(2)_L \times SU(2)_R$. We conclude that the corresponding supergravity mode will not be turned on even in an unsmeared solution.

A.2 A D3-brane on the Conifold

In this Appendix we give a detailed treatment of the toy model of §2.2, after collecting the relevant supergravity formulas in §A.2.1.

A.2.1 General strategy

We first assemble some well-known expressions pertinent for the evaluation of mass matrices for general chiral superfields. As in the bulk of the Chapter, we will work with the Kähler potential K and the superpotential W for some chiral superfields in fixed Kähler gauge. The F-term scalar potential is

$$V_F = e^{K/M_{Pl}^2}\left(K^{A\bar{B}} D_A W D_{\bar{B}} \overline{W} - 3\frac{|W|^2}{M_{Pl}^2}\right), \tag{A.9}$$

where the Kähler covariant derivative is $D_A W = (\partial_A + \frac{K_A}{M_{Pl}^2})W$. We are interested in expressions for the masses Taylor expanded around a supersymmetric point denoted Z_0, for which $F_A \equiv D_A W|_{Z_0} = 0$, for all values of the index A. The scalar mass matrices at this point are given by

$$\nabla_a \nabla_{\bar{b}} V_F|_{Z_0} = \partial_a \partial_{\bar{b}} V_F|_{Z_0} = e^{K/M_{Pl}^2}\left(K^{A\bar{B}}\partial_a(D_A W)\partial_{\bar{b}}(\bar{D}_{\bar{B}}\overline{W}) - 2\frac{|W|^2}{M_{Pl}^4}K_{a\bar{b}}\right), \tag{A.10}$$

$$\nabla_a \nabla_b V_F|_{Z_0} = \partial_a \partial_b V_F|_{Z_0} = -\frac{e^{K/M_{Pl}^2}\overline{W}}{M_{Pl}^2}\partial_a D_b W, \tag{A.11}$$

where we have used that $\partial V_F|_{Z_0} = 0$ to replace the covariant derivatives with partial derivatives. The first-order corrections from the F-term potential, $\delta Z^M \nabla_M \partial^2_{a\bar{b}} V_F$ and $\delta Z^M \nabla_M \partial^2_{ab} V_F$, are obtained by taking three derivatives on the F-term potential:[3]

$$\begin{aligned}
\partial_{ab}\partial_{\bar{c}} V_F|_{Z_0} = {} & \partial_{\{a}(e^{K/M_{Pl}^2}K^{c\bar{d}})\partial_{b\}}(F_c)\partial_{\bar{c}}(\bar{F}_{\bar{d}}) + \partial_{\{a}(e^{K/M_{Pl}^2}K^{c\bar{d}})\partial_{\bar{c}}(F_c)\partial_{b\}}\bar{F}_{\bar{d}} + \\
& + \partial_{\{a}(F_c)\partial_{b\}}(\bar{F}_{\bar{d}})\partial_{\bar{c}}(e^{K/M_{Pl}^2}K^{c\bar{d}}) + \\
& + e^{K/M_{Pl}^2}K^{c\bar{d}}\Big[\partial^2_{ab}(F_c)\partial_{\bar{c}}\bar{F}_{\bar{d}} + \partial^2_{\bar{c}\{a}(F_c)\partial_{b\}}(\bar{F}_{\bar{d}}) + \\
& + \partial_{\{a}(F_c)\partial^2_{b\}\bar{c}}(\bar{F}_{\bar{d}}) + \partial_{\bar{c}}(F_c)\partial^2_{ab}(\bar{F}_{\bar{d}})\Big] - 3\left[e^{K/M_{Pl}^2}\frac{W}{M_{Pl}^2}\partial^2_{ab}\bar{F}_{\bar{c}}\right].
\end{aligned} \tag{A.12}$$

The fermion masses are

$$m_{ab} = e^{K/2M_{Pl}^2}\left[\partial_a D_b W + \frac{K_a}{M_{Pl}^2}D_b W - \Gamma^d_{ab}D_d W\right], \tag{A.13}$$

where the Christoffel symbol is constructed out of the Kähler metric, $\Gamma^d_{ab} = K^{d\bar{c}}K_{a\bar{c}b}$. In an expansion around Z_0, we will be interested in the first-order corrections to the supersymmetric masses, which are given by

$$m_{ab}|_{Z_\star} = m_{ab}|_{Z_0} + \delta Z^M(\nabla_M m_{ab})|_{Z_0}\,, \tag{A.14}$$

[3]Since the expressions turn out to be slightly lengthy, only the terms with two holomorphic indices and one anti-holomorphic index are worked out here. To obtain the complete set of corrections it is necessary to also work out the case when all indices are holomorphic.

where M runs over both holomorphic and anti-holomorphic indices. The lowest-order contribution is then

$$m_{ab}|_{Z_0} = e^{\frac{K}{2M_{Pl}^2}} \partial_a D_b W. \tag{A.15}$$

The vanishing of the F-terms implies that $\partial_a D_b W|_{Z_0} = \partial_b D_a W|_{Z_0}$.

The gravitino mass is given by $m_{3/2}^2 = e^{K/M_{Pl}^2} \left|\frac{W}{M_{Pl}^2}\right|^2$. At the supersymmetric minimum, $m_{3/2}^2\big|_{Z_0} = \frac{|V_F|}{3M_{Pl}^2}$, and to linear order in the expansion around this minimum,

$$m_{3/2}^2 = m_{3/2}^2\Big|_{Z_0} + \delta Z^M \partial_M \left(e^{K/M_{Pl}^2} \left| \frac{W}{M_{Pl}^2} \right|^2 \right)\Bigg|_{Z_0} = m_{3/2}^2\Big|_{Z_0}, \tag{A.16}$$

where the last step follows from F-flatness at Z_0. The small mixing terms between the gravitino and the chiral fermions are proportional to $e^{K/2M_{Pl}^2} \frac{F_a}{M_{Pl}^2}$, and will henceforth be neglected.

Specializing to the KKLT scenario with Kähler potential $K = -3M_{Pl}^2 \ln U$ and an uplift potential of the form $V_{\rm up} = \frac{D}{U^2} = 3m_{3/2}^2 M_{Pl}^2$, the masses from the uplift potential can be written as

$$\partial_{MN}^2 V_{\rm up} = \frac{2V_{\rm up}}{3M_{Pl}^2} \left(K_{MN} + \frac{2}{3M_{Pl}^2} K_M K_N \right). \tag{A.17}$$

To obtain the first-order correction to the mass matrix, following the logic for corrections from V_F above, we take three derivatives of the uplift potential,

$$\partial_{MNP}^3 V_{\rm up} = \frac{2|V_F|}{3M_{Pl}^2} \left[K_{MNP} + \frac{2}{3M_{Pl}^2} (K_M K_{NP} + \text{cycl.perm.}) - \frac{16}{(3M_{Pl}^2)^2} K_M K_N K_P \right]. \tag{A.18}$$

We will discuss the role of the first-order corrections in §A.2.4. Adding the lowest-order contributions to the scalar masses from V_F and $V_{\rm up}$, we obtain

$$\partial_{a\bar{b}}^2 V_{\rm tot}|_{Z_0} = K^{c\bar{d}} m_{ac} m_{\bar{b}\bar{d}} + \frac{4}{3} \frac{m_{3/2}^2}{M_{Pl}^2} K_a K_{\bar{b}}, \tag{A.19}$$

$$\partial_{ab}^2 V_{\rm tot}|_{Z_0} = m_{3/2}^2 \left[K_{ab} + \frac{4}{3M_{Pl}^2} K_a K_b - \frac{M_{Pl}^2 W_{ab}}{W} \right]. \tag{A.20}$$

Equation (A.20) gives the lowest-order contribution to the B terms, while the mass splittings between the scalars and fermions are given by

$$M^2_{a\bar{b}}\Big|_{Z_*} = \frac{4}{3}\frac{m^2_{3/2}}{M^2_{Pl}} K_a K_{\bar{b}}\,, \tag{A.21}$$

to this order in perturbation theory.

A.2.2 Vacua for a D3-brane on the conifold

We remind the reader that the four-dimensional effective theory we are studying is given by

$$K = -3M^2_{Pl}\ln\, U = -3M^2_{Pl}\ln(T + \bar{T} - \gamma k)\,, \tag{A.22}$$

$$W = W_0 + W_{\rm np} = W_0 + \mathcal{A}_0 e^{-\xi}\,. \tag{A.23}$$

Here $\xi = aT + \zeta$, $\zeta = -\frac{1}{n}\ln f(z)$, and $k = r^2$. The number n of D7-branes determines $a = 2\pi/n$, and f is the dimensionless embedding function of the four-cycle responsible for the nonperturbative superpotential. Since we will need to be careful about the dimensions, it is worth mentioning that the volume modulus T is dimensionless, and γ has mass dimension -2. One can introduce fields with canonical dimension 1, e.g. $Z^T = \lambda T$, $Z^i = \sigma_i z_i$ where dimensionful constants $[\lambda] = 1, [\sigma_i] = -1/2$ have been introduced. These constants are fixed by the kinetic terms, by requiring canonically normalized fields at the supersymmetric minimum, as discussed in §A.2.3.

Supersymmetric AdS solution

As discussed in the bulk of the Chapter, we find supersymmetric AdS vacua by solving the F-term equations. The equation (2.26) for the volume modulus can be written as

$$W = -\frac{aU}{3} W_{\rm np}\,, \tag{A.24}$$

which, upon defining $\xi = aT + \zeta$, with $\zeta = -\frac{1}{n}\ln f(z)$, leads to an algebraic, transcendental equation for U

$$\left(1 + \frac{aU}{3}\right)^2 e^{-aU - a\gamma k - \xi - \bar{\xi}} = \left|\frac{W_0}{\mathcal{A}_0}\right|^2 . \tag{A.25}$$

Equation (2.26) also determines the axionic, imaginary part of T,

$$\Im(T) = -\frac{1}{a}\arg\left(\frac{-W_0 e^{\zeta}}{\mathcal{A}_0}\right). \tag{A.26}$$

In a chart where we use z_2, z_3 and z_4 as independent complex coordinates on the conifold and for the Kuperstein embedding $f(z) = \frac{z_2 - \mu}{\mu}$, equation (2.28) becomes

$$-\frac{1}{n(z_2 - \mu)} + \frac{2a\gamma}{3r}\left(\bar{z}_2 - \frac{\bar{z}_1 z_2}{z_1}\right) = 0\,, \tag{A.27}$$

$$\frac{2a\gamma}{3r}\left(\bar{z}_3 - \frac{\bar{z}_1 z_3}{z_1}\right) = 0\,, \tag{A.28}$$

$$\frac{2a\gamma}{3r}\left(\bar{z}_4 - \frac{\bar{z}_1 z_4}{z_1}\right) = 0\,, \tag{A.29}$$

after using that $k = r^2$ far from the tip of the conifold, and that $\partial_i z_1 = -\frac{z_i}{z_1}$ for $i = 2, 3, 4$ in this chart. The radius is related to the standard complex coordinates on the conifold by $\sum_{a=1}^{4} |z_a|^2 = r^3$. Writing $z_A = |z_A| e^{i\eta_A}$, equations (A.28, A.29) imply that $\eta_1 = \eta_3 = \eta_4$, but they do not restrict the norms of z_3, z_4. Equation (A.27) on the other hand can be written as

$$\frac{1}{(|z_2| \pm |\mu|)^3} = 4\left(\frac{4\pi\gamma}{3}\right)^3 |z_2|\,. \tag{A.30}$$

The different signs come from choosing either $\eta_2 = \eta_\mu$ for the upper sign or $\eta_2 = \eta_\mu + \pi$ for the lower, reflecting the fact that there are two distinct supersymmetric loci for the D3-brane: one located just *above* the D7-branes, and the other located far down the throat. Since the norms of z_3 and z_4 are undetermined, we have a two-dimensional moduli space. We choose to do our analysis for the point where $z_3 = z_4 = 0$ and z_2 is real by choice of the phase of μ.

Other D7-brane embeddings

As an illustration of the fact that our supersymmetric solutions are generic for a large class of D7-brane embeddings, we derive the kindred solution for the Ouyang embedding [92] specified by the embedding function $f(w) = \frac{w_2-\mu}{\mu}$ when written in terms of the w-coordinates of equation (A.2). The solution has a moduli space consisting of two isolated points satisfying the equation

$$\frac{\omega}{\omega \pm \mu} = \frac{2an\gamma}{3}\omega^{4/3}\,, \tag{A.31}$$

One of these solutions is located at $\omega \gtrsim \mu$, while the other is at $\omega \gtrsim 0$. In this notation $w_2 = \omega e^{i\eta}$, where the phase η is fixed to be the phase of μ — possibly up to a phase difference π, and here $r = \omega^{2/3}$. This illustrates that supersymmetric solutions are generic, and that the pairing of solutions that we have commented upon may be a feature of a wide variety of D7-brane embeddings.

A.2.3 Canonical Normalization

In order to correctly assess the scaling of the masses, we obtain the canonically normalized fields. With the Kähler potential (A.22), the Kähler metric is given by

$$K_{a\bar{b}} = 3M_{Pl}^2\left[\frac{U_a U_{\bar{b}}}{U^2} - \frac{U_{a\bar{b}}}{U}\right]. \tag{A.32}$$

We find that, in our case, the diagonalization of this metric is essentially captured by choosing the constants λ, σ_i appropriately:

$$\lambda := \left(\frac{3M_{Pl}^2}{U^2}\right)^{1/2} , \tag{A.33}$$

$$\sigma_2 := \left(\frac{M_{Pl}^2}{U}\left[\frac{B}{\pi\mu_0^2}\right]\right)^{1/2} , \tag{A.34}$$

$$\sigma_3 := \left(\frac{3M_{Pl}^2}{U}\left[\frac{B}{4\pi\mu_0^2}\right]\right)^{1/2} . \tag{A.35}$$

This normalization gives $K_{T\bar{2}} = -\frac{\sqrt{3}}{2(\pi UB)^{1/2}}$. Thus, this entry is suppressed by $O(\frac{1}{\sqrt{UB}})$ with respect to the other entries in the metric, and does not affect the determinant of the metric to the order that we are working. A completely diagonal metric may be chosen by performing a unitary transformation after specifying the constants λ, σ. However, encouraged by the relative smallness of the off-diagonal metric elements — for the values of U and B discussed in §2.2.3, the off-diagonal metric elements are of order 10^{-3} — we will not perform this unitary transformation that would mix the hidden sector Kähler modulus with our proxy D3-brane visible fields. To be explicit, taking two derivatives on the Kähler potential and evaluating it at the supersymmetric point with this definition of λ and σ_i for the canonically normalized fields gives

$$K_{MN} = \delta_{MN}^{T\bar{T}, z_2\bar{z}_2, z_3\bar{z}_3, z_4\bar{z}_4} - \frac{\sqrt{3}}{2(\pi UB)^{1/2}}\left(\delta_{\{MN\}}^{T\bar{2}} + \delta_{\{MN\}}^{\bar{T}2}\right) + \tag{A.36}$$

$$+\left[\delta_{MN}^{TT,33,44} - \frac{1}{2}\delta_{MN}^{22} - \frac{\sqrt{3}}{2(\pi UB)^{1/2}}\delta_{\{MN\}}^{T2} + c.c.\right] . \tag{A.37}$$

Here, a δ-function with two indices is a shorthand for two delta functions. The curly braces correspond to symmetrization, without a factor of $\frac{1}{2}$, i.e. $\delta_{\{MN\}}^{PQ} = \delta_M^P\delta_N^Q + \delta_N^P\delta_M^Q$. As usual, only the holomorphic+antiholomorphic derivatives correspond to metric elements. The inverse Kähler metric is given, to leading order in $1/B$ and to second order in $1/(aU)$, by $K^{a\bar{b}} = \delta^{a\bar{b}} + \frac{\sqrt{3}}{2(\pi UB)^{1/2}}\delta_{T\bar{2},2\bar{T}}^{a\bar{b}}$. It is well-known and easy to verify that the Kähler metric is no-scale, $K^{A\bar{B}}K_AK_{\bar{B}} = 3M_{Pl}^2$, and that $K^{A\bar{B}}K_A = -\lambda U\ \delta_{\bar{T}}^{\bar{B}}$, by using

$$K_M = -\sqrt{3}M_{Pl}\delta_M^T + \frac{3M_{Pl}}{2(\pi UB)^{1/2}}\delta_M^2 + c.c. \tag{A.38}$$

A.2.4 Details of the mass matrix

Using equation (A.37) and the fact that

$$\partial_a D_b W = -\frac{W}{M_{Pl}^2}\left((aU+2)\delta_{ab}^{TT} + \frac{1}{2}\delta_{ab}^{22} - \delta_{ab}^{33,44} - \frac{\sqrt{3}(aU+2)}{2(\pi UB)^{1/2}}\delta_{\{ab\}}^{T2}\right), \tag{A.39}$$

the AdS supersymmetric masses (A.10) and (A.11) are easily evaluated. We find that in our case, the AdS supersymmetric B terms, denoted $\mathcal{B}_{ab}$ to distinguish them from the Minkowski space B term, turn out to be real. Thus, the mass matrix separates into two blocks when written in terms of real fields, $Z^a = X^a + iY^a$, as $V_F \supset (\mathcal{M}_{\rm tot}^2)_{a\bar{b}}Z^a\bar{Z}^{\bar{b}} + \frac{1}{2}(\mathcal{B}_{ab}Z^aZ^b + h.c.) = \mathcal{M}_{X^aX^b}^2X^aX^b + \mathcal{M}_{Y^aY^b}^2Y^aY^b$, with

$$\mathcal{M}_{X^aX^b}^2 = \left((\mathcal{M}_{\rm tot}^2)_{a\bar{b}} + \mathcal{B}_{ab}\right), \tag{A.40}$$

$$\mathcal{M}_{Y^aY^b}^2 = \left((\mathcal{M}_{\rm tot}^2)_{a\bar{b}} - \mathcal{B}_{ab}\right). \tag{A.41}$$

Here $\mathcal{B}_{ab} = \partial_a\partial_b V_F|_{Z_0}$, and $(\mathcal{M}_{\rm tot}^2)_{a\bar{b}} = \partial_{a\bar{b}}V_F|_{Z_0}$ denotes the total AdS scalar mass. After the canonical normalization discussed in §A.2.3, the resulting scalar mass matrices are most transparently written in terms of real fields for which we have — to leading order in $1/B$ and to second order in $1/(aU)$ — the supersymmetric masses:

$$\partial_{MN}^2 V_F\,|_{Z_0} =$$

$$
= \begin{pmatrix}
m^2_{T\bar{T}}+m^2_{TT} & m^2_{T\bar{2}}+m^2_{T2} & & & & & & \\
m^2_{T\bar{2}}+m^2_{T2} & m^2_{2\bar{2}}+m^2_{22} & & & & & & \\
 & & m^2_{3\bar{3}}+m^2_{33} & & & & & \\
 & & & m^2_{4\bar{4}}+m^2_{44} & & & & \\
 & & & & m^2_{T\bar{T}}-m^2_{TT} & m^2_{T\bar{2}}-m^2_{T2} & & \\
 & & & & m^2_{T\bar{2}}-m^2_{T2} & m^2_{2\bar{2}}-m^2_{22} & & \\
 & & & & & & m^2_{3\bar{3}}-m^2_{33} & \\
 & & & & & & & m^2_{4\bar{4}}-m^2_{44}
\end{pmatrix}
\tag{A.42}
$$

$$
= \frac{|V_F|}{2M_{Pl}^2}
\begin{pmatrix}
\frac{2}{3}(a^2U^2+5aU) & -\frac{a^2U^2+5aU}{\sqrt{3}(\pi UB)^{\frac{1}{2}}} & & & & & & \\
-\frac{a^2U^2+5aU}{\sqrt{3}(\pi UB)^{\frac{1}{2}}} & \frac{a^2U}{2\pi B}-\frac{5}{6} & & & & & & \\
 & & -\frac{4}{3} & & & & & \\
 & & & -\frac{4}{3} & & & & \\
 & & & & \frac{2}{3}(a^2U^2+3aU) & -\frac{a^2U^2+3aU}{\sqrt{3}(\pi UB)^{\frac{1}{2}}} & & \\
 & & & & -\frac{a^2U^2+3aU}{\sqrt{3}(\pi UB)^{\frac{1}{2}}} & \frac{a^2U}{2\pi B}-\frac{3}{2} & & \\
 & & & & & & 0 & \\
 & & & & & & & 0
\end{pmatrix} . \tag{A.43}
$$

Here $V_F = V_F\big|_{Z_0} = \Lambda_{\text{AdS}} M_{Pl}^2 = -3m_{3/2}^2 M_{Pl}^2$. From this expression, it is evident that there are several tachyonic directions at this AdS vacuum. The stability of the solution requires masses larger than the Breitenlohner-Freedman mass, which in AdS_4 is $\mathcal{M}^2_{\text{BF}} = -\frac{3}{2}\frac{|V_F|}{M_{Pl}^2}$. The eigenvalues of matrix (A.43) are $\frac{|V_F|}{M_{Pl}^2}\left[\frac{1}{3}(a^2U^2+5aU), -\frac{5}{12}, -\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}(a^2U^2+3aU), -\frac{3}{4}, 0, 0\right]$, to leading order in $1/B$ and to second order in $1/aU$, so as expected there is no instability. The supersymmetric mass-splittings can be read off from equation (2.46),

$$
\mathcal{M}^2_{a\bar{b}}\Big|_{Z_0} = -2e^{K/M_{Pl}^2}\frac{|W|^2}{M_{Pl}^4}K_{a\bar{b}} = -2\, m_{3/2}^2 K_{a\bar{b}} \tag{A.44}
$$

at the AdS supersymmetric point, while the B term masses are proportional to the fermion masses,

$$\mathcal{B}_{ab}|_{Z_0} = \partial^2_{ab} V_F\Big|_{Z_0} = -\frac{e^{K/M_{Pl}^2}\overline{W}}{M_{Pl}^2}\partial_a D_b W \tag{A.45}$$

$$= \frac{|V_F|}{3M_{Pl}^2}\left\{(aU+2)\delta^{TT}_{ab} + \frac{1}{2}\delta^{22}_{ab} - \delta^{33,44}_{ab} - \frac{\sqrt{3}(aU+2)}{2(\pi U B)^{1/2}}\delta^{T2}_{\{ab\}}\right\}. \tag{A.46}$$

Mass matrix after uplift

After incorporating the supersymmetry breaking by adding the uplift potential to the F-term potential, the vacuum expectation values of the moduli get slightly modified. In this section we confirm that this shift in vevs is indeed small, and we demonstrate the surprising fact that the D3-brane does not move upon uplifting to this order in perturbation theory. To compute the shift we need the inverse of the total mass matrix at the supersymmetric point, [4]

$$\delta Z^M = -\left(\partial_N \partial_M (V_F + V_{\rm up})\right)^{-1}\Big|_{Z_0} \partial_N V_{up}\Big|_{Z_0}. \tag{A.47}$$

It is easy to see that $\partial_N V_{\rm up} = \frac{2}{3}\frac{|V_F|}{M_{Pl}^2} K_N = 2m^2_{3/2} K_N$, while two derivatives on the uplift potential can be written as in equation (2.42). Together with the contribution from the F-term potential and expressed in the real basis, the full mass matrix at Z_0 is

$$\partial^2_{MN}\left(V_{\rm tot}\right)\big|_{Z_0} =$$

[4]This formula applies to all coordinates except Y^3 and Y^4. In these directions there is no shift to this order by the vanishing of the mass matrix and first derivative on the uplift potential.

$$
= \frac{|V_F|}{2M_{Pl}^2}\begin{pmatrix}
\frac{2}{3}(a^2U^2+5aU) & -\frac{a^2U^2+5aU}{\sqrt{3}(\pi UB)^{\frac{1}{2}}} & & & & & & \\
-\frac{a^2U^2+5aU}{\sqrt{3}(\pi UB)^{\frac{1}{2}}} & \frac{a^2U}{2\pi B}-\frac{1}{6} & & & & & & \\
 & & +\frac{4}{3} & & & & & \\
 & & & +\frac{4}{3} & & & & \\
 & & & & \frac{2}{3}(a^2U^2+3aU) & -\frac{a^2U^2+3aU}{\sqrt{3}(\pi UB)^{\frac{1}{2}}} & & \\
 & & & & -\frac{a^2U^2+3aU}{\sqrt{3}(\pi UB)^{\frac{1}{2}}} & \frac{a^2U}{2\pi B}+\frac{1}{2} & & \\
 & & & & & & 0 & \\
 & & & & & & & 0
\end{pmatrix}. \tag{A.48}
$$

Inverting, we find that to this order the only shift is in the real part of T and is given by $\frac{1}{2}(\delta Z^T + \delta\bar{Z}^{\bar{T}}) = \frac{\sqrt{3}M_{Pl}}{aU(aU+5)}$.

To order $1/(aU)^2, 1/B$ the fermion mass matrix at Z_0 is obtained by evaluating (A.15),

$$
m_{ab}\Big|_{Z_0} = e^{K/(2M_{Pl}^2)}\frac{W}{M_{Pl}^2}\left\{-(aU+2)\delta_{ab}^{TT} - \frac{1}{2}\delta_{ab}^{22} + \delta_{ab}^{33,44} + (aU+2)\frac{\sqrt{3}}{2(\pi UB)^{1/2}}\delta_{\{ab\}}^{T2}\right\}. \tag{A.49}
$$

The prefactor $e^{K/2M_{Pl}^2}\frac{W}{M_{Pl}^2}$ is just $m_{3/2}\big|_{Z_0}$. The gravitino mass is unchanged to this order from its value in AdS,

$$
m_{3/2}^2 = \frac{|V_F||_{Z_0}}{3M_{Pl}^2}\,. \tag{A.50}
$$

The mass splittings between the scalars and fermions in the Minkowski solution are, to this order,

$$
M_{a\bar{b}}^2|_{Z_\star} = \mathcal{M}_{a\bar{b}}^2|_{Z_0} + \partial_{a\bar{b}}^2 V_{\rm up}|_{Z_0} = \tag{A.51}
$$

$$
= 4m_{3/2}^2\begin{pmatrix}
1 & -\frac{\sqrt{3}}{2(\pi UB)^{1/2}} & & \\
-\frac{\sqrt{3}}{2(\pi UB)^{1/2}} & \frac{3}{4\pi UB} & & \\
 & & 0 & \\
 & & & 0
\end{pmatrix}. \tag{A.52}
$$

The B terms are, to this order,

$$B_{ab} \equiv \partial_{ab} V_{\text{tot}}|_{Z_\star} = \mathcal{B}_{ab}|_{Z_0} + \partial_{ab} V_{\text{up}}|_{Z_0} = \tag{A.53}$$

$$= m_{3/2}^2 \begin{pmatrix} aU & -\frac{\sqrt{3}a}{2}(\frac{U}{\pi B})^{1/2} & & \\ -\frac{\sqrt{3}a}{2}(\frac{U}{\pi B})^{1/2} & -\frac{1}{2} & & \\ & & 1 & \\ & & & 1 \end{pmatrix} . \tag{A.54}$$

Corrections to the mass matrix due to the shift in the volume modulus

We determine the relative importance of the different contributions by computing the first-order correction to the mass matrix in perturbation theory. The contribution from the uplift potential comes from evaluation of equation (A.18). Three derivatives on the Kähler potential, evaluated at Z_0, can be written as

$$\begin{aligned} K_{MNP}|_{Z_0} = \frac{1}{M_{Pl}} \Bigg\{ & (\pi U B)^{1/2} \left(\frac{2}{9}\delta^{222}_{MNP} + \frac{2}{3}\delta^{33\bar{2},44\bar{2}}_{MNP} - \frac{4}{3}\delta^{332,442}_{MNP} - \frac{1}{3}\delta^{22\bar{2}}_{MNP} - \frac{1}{3}\delta^{3\bar{3}2,4\bar{4}2}_{\{MN\}P} \right) - \\ & -\frac{1}{\sqrt{3}} \left(\delta^{2\bar{2}T,3\bar{3}T,4\bar{4}T}_{\{MN\}P} + \frac{2}{3}\delta^{TTT}_{MNP} + 2\delta^{TT\bar{T}}_{MNP} - \frac{1}{2}\delta^{22T}_{MNP} - \frac{1}{2}\delta^{22\bar{T}}_{MNP} + \delta^{33T,44T}_{MNP} + \delta^{33\bar{T},44\bar{T}}_{MNP} \right) + \\ & + \frac{1}{2(\pi U B)^{1/2}} \left[\frac{5}{2}\delta^{TT2}_{MNP} + 2\delta^{TT\bar{2}}_{MNP} + 2\delta^{T\bar{T}2}_{\{MN\}P} \right] + c.c. + \text{perm}_{(M,N)\leftrightarrow P} \Bigg\} . \end{aligned} \tag{A.55}$$

After taking $\text{perm}_{(M,N)\leftrightarrow P}$ into account we read off that e.g. $K_{222} = \frac{2\sqrt{\pi U B}}{3M_{Pl}}$. Remembering that the shift is only in the direction of the volume modulus, we can immediately estimate the size of the contributions to the mass matrix from equation (A.18). Recall from equations (A.37, A.38) that K_{MN} is no larger than $\mathcal{O}(1)$ and that K_M is no larger than $\mathcal{O}(1)$. With one index being canonically normalized T or $\bar{T}$, equation (A.55) gives that K_{MNP} is no larger than $\mathcal{O}(1)$. All together, $\partial^3 V_{\text{up}}$ is no larger than $\mathcal{O}(\frac{|V_F|}{M_{Pl}^3})$, but the shift in the real part of the Kähler modulus scales like $\delta X^T \sim \frac{M_{Pl}}{(aU)^2}$. The contribution to the mass matrix from the uplift potential will therefore scale like $\delta X^T\ \partial^3 V_{\rm up}\big|_{Z_0} \sim \frac{|V_F|}{M_{Pl}^2}\frac{1}{(aU)^2}$. The smallest non-vanishing entry in $\partial^2 V_{\rm tot}\big|_{Z_0} \sim 1$, so we conclude that the first-order correction from the uplift potential will come in at a subleading order in $1/(aU)$ and can consistently be dropped. By direct evaluation we find that $\delta m^2_{3\bar{3}} = \delta B_{33}$, which means that the flat directions are not lifted by the uplift potential.

The first-order correction from the F-term potential is more tedious to obtain, but follows from straightforward evaluation of equation (A.12) and the corresponding equation for all holomorphic indices. These terms also do not contribute before order $1/aU$.

Finally, the magnitude of the F-terms for the canonically normalized fields can be found to linear order by contracting equation (A.39) with the shift (A.47).

A.2.5 A bound on B

Requiring that both the D3-brane and D7-brane are located in the warped region gives a bound on B. Since we are considering only solutions in which the D3-brane lies deeper down in the throat than the D7-brane, B is bounded from below: $B > 1$. The upper bound comes from considering the arguments in [69] and [74], in which a bound on k is obtained in terms of N, the number of D3-branes that make up the throat before we add our toy visible sector:

$$\frac{\gamma k}{T+\bar{T}} \leq \frac{2}{3}\frac{1}{N}\,. \tag{A.56}$$

For the D7-brane to extend down the throat, the bound (A.56) should apply if evaluated at the point of lowest descent of the D7-brane into the throat. We have

$\gamma k_{\mathrm{D7}} = \frac{3}{4\pi} B^{1/3}$, which follows from direct evaluation of B. It follows that

$$B \leq \left(\frac{8\pi}{9}\right)^3 \left(\frac{T+\bar{T}}{N}\right)^3 . \tag{A.57}$$

In this case $U = T + \bar{T} - \gamma k_{\mathrm{D7}} = T + \bar{T} - \frac{3}{4\pi} B^{1/3}$. If $4^3 U^3 \gg B$, then the above bound can be written as

$$B \leq L^3 . \tag{A.58}$$

Together these bounds imply that $1 < B \leq L^3$. In particular, a consistent solution requires that $L > 1$, from which it follows that

$$L = \frac{8\pi}{9} \frac{U}{N} \approx 2.79 \frac{U}{N} > 1 . \tag{A.59}$$

APPENDIX B

APPENDIX FOR CHAPTER 4

B.1 The distribution of critical points

In this appendix we briefly review some pertinent results from the the study of the distribution of non-supersymmetric vacua by Denef and Douglas [33]. In particular, we will review how, for any fixed cosmological constant, the density of *critical points* — stable and unstable — grows linearly with F towards the boundary of the approximately-supersymmetric regime. This gives evidence for the expectation that a "generic critical point" typically does not exhibit any particular hierarchy between $m_{\rm susy}$ and F, i.e. typical critical points in random supergravity are not predominantly of the approximately supersymmetric kind. We also comment on how the distribution of metastable vacua is modified by the exponential suppression of the probability density found in §4.5.

The density of critical points with cosmological constant $\langle V\rangle = v$ can be evaluated from

$$N_{\rm crit.pts.}(v) = \int d\mu[W, F, Z, U]\, \delta^{2N}(\partial V) \left|\det\mathcal{H}\right|\; \delta(V - v)\,. \tag{B.1}$$

Just as in reference [33], we assume flat prior probabilities for W, F, Z and U between 0 and a cutoff Λ. Although this assumption is made here for simplicity, interesting domains of the string theory landscape have been argued to be well-described by these priors. The measure used in this appendix is

$$d\mu[W, F, Z, U] = C\, d^2W\, d^{2N}F\, d^{k_Z}Z\, d^{k_U}U\ \ , \tag{B.2}$$

where $k_Z = N(N+1)$, $k_U = \frac{N}{3}(N+1)(N+2)$, and C is a normalization constant.

As reviewed in §4.2, the critical point equation can be written as an eigenvalue equation for the matrix $\mathcal{M}$ of equation (4.6), enforcing that $\mathcal{M}$ has an eigenvalue equal to $2|W|$ with $\hat{F}$ being proportional to the corresponding eigenvector. Thus, the critical point equation can be simplified by expressing the integration over Z_{ab} (and thereby $\mathcal{M}$) as an integral over the ordered eigenvalues, $\lambda_1 \leq \ldots \leq \lambda_N$, and unitary rotations $\mathcal{U}$. Let us denote an orthonormal eigenbasis of $\mathcal{M}$ as $e_a^{\pm}$, with corresponding eigenvalues $\pm\lambda_a$. In this basis $\hat{F}$ has components $|F|\eta_a^{\pm}$, with $\sum_{a=1}^{N}\left((\eta_a^+)^*\eta_a^+ + (\eta_a^-)^*\eta_a^-\right) = 1$. In this notation,

$$\begin{aligned}
\delta^{2N}(\partial V) &= \delta^{2N}\Big((M - 2|W|)\hat{F}\Big) = \frac{1}{|F|^{2N}} \prod_{a=1}^{N} \delta\Big(\eta_a^+(\lambda_a - 2|W|)\Big)\, \delta\Big(\eta_a^-(-\lambda_a - 2|W|)\Big) \\
&= \frac{1}{|F|^{2N}} \prod_{a=1}^{N} \delta\Big(\eta_a^+(\lambda_a - 2|W|)\Big)\, \frac{\delta(\eta_a^-)}{\lambda_a + 2|W|} \,. \qquad \text{(B.3)}
\end{aligned}$$

The cosmological constant constraint $\delta(V - v)$ can be written as

$$\delta(V - v) = \delta(F^2 - 3|W|^2 - v) = \frac{\delta(|W| - w)}{3(|W| + w)}\,, \qquad \text{(B.4)}$$

where $w^2 = \frac{1}{3}\left(F^2 - v\right)$. Although the integral (B.1) can also be estimated in the generic regime in which $F \sim m_{\text{susy}}$, this evaluation is slightly technical, and for the purpose of this appendix it suffices to discuss the approximately-supersymmetric regime of Denef and Douglas [33]. In this case, the integration over F is cut off before $F = m_{\text{susy}}/N$, and the determinant of the Hessian appearing in the integrand of equation (B.1) is well-approximated by

$$|\det\mathcal{H}| \approx m_+^2\, m_-^2 \prod_{a'=2}^{N} \left(\lambda_{a'}^2\right)^2\,, \qquad \text{(B.5)}$$

with $m_{\pm}^2$ as in equation (4.71). The number of critical points is

$$\begin{aligned}
N_{\text{crit.pts.}}(v) = C \int d\mu[\vartheta_F, \Omega_F, U, \mathcal{U}] \int_0^{\Lambda^2} d|W|^2 \int_0^{\epsilon\, m_{\text{susy}}} \frac{dF}{F}\, \frac{\delta(|W| - w)}{3(|W| + w)} \times \\
\times\left[\prod_{a=1}^{N} \int_{\lambda_{a-1}}^{\lambda_{a+1}} d\lambda_a\, \delta\Big(\eta_a^+(\lambda_a - 2|W|)\Big)\, \frac{\delta(\eta_a^-)}{\lambda_a + 2|W|}\right] f(\lambda_1, \ldots, \lambda_N)\, |\det\mathcal{H}|\,, \qquad \text{(B.6)}
\end{aligned}$$

where $f(\lambda_1, \ldots, \lambda_N)$ denotes the joint probability density of equation (4.39), ϵ is a small number, and for notational convenience we have defined $\lambda_0 = 0$ and $\lambda_{N+1} = \Lambda$. In the approximately-supersymmetric regime, only λ_1 has a non-negligible probability density at $2|W|$. With this observation, the integral simplifies to

$$N_{\text{crit.pts.}}(v) = C \int d\mu[\vartheta_F, \Omega_F, U, \mathcal{U}] \Big(\prod_{a'=2}^{N} \delta^N(\eta^+_{a'}) \Big) \Big(\prod_{a=1}^{N} \delta^N(\eta^-_a) \Big) \int_0^{\epsilon' \lambda_2} \frac{dF}{F} \frac{w}{6w} \times$$
$$\times \int_0^{\lambda_2} d\lambda_1 \frac{\delta(\lambda_1 - 2w)}{\lambda_1 + 2w} \left[\prod_{a'=2}^{N} \int_{\lambda_{a'-1}}^{\lambda_{a'+1}} d\lambda_a \, \frac{1}{\lambda_{a'}^2 - 4w^2} \right] f(\lambda_1 = 2w, \ldots, \lambda_N) \, |\det \mathcal{H}| \tag{B.7}$$

where now the constant $\epsilon' < 1$ encodes the assumed hierarchy between $\lambda_{a'}$ and F. Since the probability density of the smallest eigenvalue exhibits a linear cleft for small arguments, cf. equation (4.40), we can (heuristically) write $f(\lambda_1 = 2w, \ldots, \lambda_N) = 2kw \, \tilde{f}(\lambda_2, \ldots, \lambda_N)$, where k is an $O(1)$ constant. This simplifies the integral to

$$N_{\text{crit.pts.}}(v) \approx \frac{k\,C}{12} \int d\mu[\vartheta_F, \Omega_F, U, \mathcal{U}] \Big(\prod_{a'=2}^{N} \delta^N(\eta^+_{a'}) \Big) \Big(\prod_{a=1}^{N} \delta^N(\eta^-_a) \Big) \times$$
$$\times \int_0^{\epsilon' \lambda_2} \frac{dF}{F} \, m_+^2 \, m_-^2 \, \left[\prod_{a'=2}^{N} \int_{\lambda_{a'-1}}^{\lambda_{a'+1}} d\lambda_a \right] \tilde{f}(\lambda_2, \ldots, \lambda_N) \, |\det \mathcal{H}'|^{1/2} \,, \tag{B.8}$$

where $\mathcal{H}'$ denotes the truncation of $\mathcal{H}$ to exclude the Goldstino direction. The scaling of the number of critical points with F at fixed cosmological constant is evidently determined by the factor

$$\int_0^{\epsilon' \lambda_2} \frac{dF}{F} \, m_+^2 \, m_-^2 \,. \tag{B.9}$$

For typical values of $U_{111} \sim m_{\text{susy}}$, the Goldstino masses-squared $m_\pm^2$ are each of order F, and the number of critical points scales with F as

$$N_{\text{crit.pts.}}(v) \sim \int_0^{\epsilon \, m_{\text{susy}}} dF \; F \,, \tag{B.10}$$

and thus, for any given scale of the supersymmetric masses, the critical points are more numerous towards the upper edge of the domain of approximate supersymmetry.

We conclude with some simple remarks. This scaling of the number of critical points with F is consistent with the computation by Denef and Douglas of the scaling of metastable vacua with F,

$$N_{\text{vacua}} \sim \int_0^{\epsilon\, m_{\text{susy}}} F^5 \, dF \,, \tag{B.11}$$

which is not surprising since the above computation closely mimics that of [33]. The different scalings of the number of critical points and the number of vacua can be understood from the additional fine-tuning necessary to obtain stability. In the approximately-supersymmetric regime it is necessary to tune $|U_{111}| \lesssim O(F)$, which gives an additional factor of F^2 from the measure $d|U_{111}|\,|U_{111}|$. Furthermore, as reviewed in §4.5, the intent of this fine-tuning is to lower the scale of the Goldstino mass-squared to $O(F^2)$ in order to improve the probability of positivity of $\mathcal{H}$. By equation (B.9), this provides two more powers of F, from which equation (B.11) follows.

Finally, with these flat priors the additional N-dependent (but F-independent) fine-tuning explored in this Chapter modifies the density of non-supersymmetric vacua in the approximately supersymmetric regime by

$$\frac{\prod_{a'=2}^{N^\alpha} \int_0^{\frac{m_{\text{susy}}}{N}} d|U_{11a'}|\,|U_{11a'}|}{\prod_{a'=2}^{N^\alpha} \int_0^{\frac{m_{\text{susy}}}{\sqrt{N}}} d|U_{11a'}|\,|U_{11a'}|} \sim e^{-N^\alpha \ln N} \,, \tag{B.12}$$

where N^α, with $\alpha \le 1$, parameterizes the number of terms in $\mathcal{S}$ of equation (4.74) that need to be fine-tuned in order for a fluctuation to positivity of the smallest eigenvalue to become likely.

BIBLIOGRAPHY

[1] G. Aad *et al.* [ATLAS Collaboration], "Combined search for the Standard Model Higgs boson using up to 4.9 fb-1 of pp collision data at sqrt(s) = 7 TeV with the ATLAS detector at the LHC," Phys. Lett. B **710**, 49 (2012) [arXiv:1202.1408 [hep-ex]]. G. Aad *et al.* [ATLAS Collaboration], "Search for the Standard Model Higgs boson in the diphoton decay channel with 4.9 fb-1 of pp collisions at sqrt(s)=7 TeV with ATLAS," Phys. Rev. Lett. **108**, 111803 (2012) [arXiv:1202.1414 [hep-ex]]. G. Aad *et al.* [ATLAS Collaboration], "Search for the Standard Model Higgs boson in the decay channel H-¿ZZ(*)-¿4l with 4.8 fb-1 of pp collision data at sqrt(s) = 7 TeV with ATLAS," Phys. Lett. B **710**, 383 (2012) [arXiv:1202.1415 [hep-ex]]. S. Chatrchyan *et al.* [CMS Collaboration], "Combined results of searches for the standard model Higgs boson in pp collisions at sqrt(s) = 7 TeV," arXiv:1202.1488 [hep-ex]. S. Chatrchyan *et al.* [CMS Collaboration], "Search for the standard model Higgs boson decaying to a W pair in the fully leptonic final state in pp collisions at sqrt(s) = 7 TeV," arXiv:1202.1489 [hep-ex]. S. Chatrchyan *et al.* [CMS Collaboration], "Search for the standard model Higgs boson decaying into two photons in pp collisions at sqrt(s)=7 TeV," arXiv:1202.1487 [hep-ex]. S. Chatrchyan *et al.* [CMS Collaboration], "Search for a Higgs boson in the decay channel H to ZZ(*) to q qbar l-l+ in pp collisions at sqrt(s) = 7 TeV," arXiv:1202.1416 [hep-ex].

[2] M. Berg, D. Marsh, L. McAllister and E. Pajer, "Sequestering in String Compactifications," JHEP **1106**, 134 (2011) [arXiv:1012.1858 [hep-th]].

[3] D. Marsh, "Towards Constraining Affleck-Dine Baryogenesis," JHEP **05**, 041 (2012) [arXiv:1108.4687 [hep-th]].

[4] D. Marsh, L. McAllister and T. Wrase, "The Wasteland of Random Supergravities," JHEP **1203**, 102 (2012) [arXiv:1112.3034 [hep-th]].

[5] K. G. Wilson, Phys. Rev. **B4** (1971) 3174, 3184.

[6] M. E. Peskin and D. V. Schroeder, "An Introduction to quantum field theory," Reading, USA: Addison-Wesley (1995) 842 p
J. Polchinski, "Effective field theory and the Fermi surface," In *Boulder 1992, Proceedings, Recent directions in particle theory* 235-274, and Calif. Univ. Santa Barbara - NSF-ITP-92-132 (92,rec.Nov.) 39 p. (220633) Texas Univ. Austin - UTTG-92-20 (92,rec.Nov.) 39 p [hep-th/9210046]. M. J. Strassler, "An Unorthodox introduction to supersymmetric gauge theory," hep-th/0309149.

[7] A. Zee (1973) Phys. Rev. **D 7**, 3630. A. Zee (1973) Phys. Rev. **D 8**, 4038. S. R. Coleman and D. J. Gross (1973), Phys. Rev. Lett. **31**, 851.

[8] J. Terning, "Modern supersymmetry: Dynamics and duality," (International series of monographs on physics. 132)

[9] J. Wess and J. Bagger, "Supersymmetry and supergravity," *Princeton, USA: Univ. Pr. (1992) 259 p.*

[10] S. P. Martin, "A Supersymmetry Primer," arXiv:hep-ph/9709356.

[11] S. R. Coleman and J. Mandula, Phys. Rev. **159** (1967) 1251.

[12] R. Haag, J. T. Lopuszanski, and M. Sohnius, Nucl. Phys. **B 88** (1975) 257.

[13] E. J. Martinec, "Nonrenormalization Theorems And Fermionic String Finiteness," Phys. Lett. B **171**, 189 (1986). M. Dine and N. Seiberg, "Nonrenormalization Theorems in Superstring Theory," Phys. Rev. Lett. **57**, 2625 (1986).

[14] Z. Komargodski and N. Seiberg, "Comments on the Fayet-Iliopoulos Term in Field Theory and Supergravity," JHEP **0906**, 007 (2009) [arXiv:0904.1159 [hep-th]].

[15] J. Polchinski, "String theory. Vol. 1: An introduction to the bosonic string," Cambridge, UK: Univ. Pr. (1998) 402 p J. Polchinski, "String theory. Vol. 2: Superstring theory and beyond," Cambridge, UK: Univ. Pr. (1998) 531 p

[16] M. B. Green, J. H. Schwarz and E. Witten, "Superstring Theory. Vol. 1: Introduction," Cambridge, Uk: Univ. Pr. (1987) 469 P. (Cambridge Monographs On Mathematical Physics) M. B. Green, J. H. Schwarz and E. Witten, "Superstring Theory. Vol. 2: Loop Amplitudes, Anomalies And Phenomenology," Cambridge, Uk: Univ. Pr. (1987) 596 P. (Cambridge Monographs On Mathematical Physics)

[17] K. Becker, M. Becker and J. H. Schwarz, "String theory and M-theory: A modern introduction," Cambridge, UK: Cambridge Univ. Pr. (2007) 739 p

[18] E. Kiritsis, "String theory in a nutshell," Princeton University Press, 2007

[19] L. McAllister, "Lectures on string theory," unpublished.

[20] R. C. Myers, "Dielectric branes," JHEP **9912**, 022 (1999) [hep-th/9910053].

[21] P. Candelas and X. de la Ossa, "Moduli Space Of Calabi-yau Manifolds," Nucl. Phys. B **355**, 455 (1991).

[22] M. Dine and N. Seiberg, "Is the Superstring Weakly Coupled?," Phys. Lett. B **162**, 299 (1985). M. Dine and N. Seiberg, "Couplings and Scales in Superstring Models," Phys. Rev. Lett. **55**, 366 (1985).

[23] J. M. Maldacena and C. Nũnez, "Supergravity description of field theories on curved manifolds and a no go theorem," Int. J. Mod. Phys. A **16**, 822 (2001) [hep-th/0007018].

[24] S. B. Giddings, S. Kachru and J. Polchinski, "Hierarchies from fluxes in string compactifications," Phys. Rev. D **66**, 106006 (2002) [hep-th/0105097].

[25] S. Gukov, C. Vafa and E. Witten, "CFT's from Calabi-Yau four folds," Nucl. Phys. B **584**, 69 (2000) [Erratum-ibid. B **608**, 477 (2001)] [hep-th/9906070].

[26] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, "De Sitter vacua in string theory," Phys. Rev. D **68**, 046005 (2003) [hep-th/0301240].

[27] V. Balasubramanian, P. Berglund, J. P. Conlon and F. Quevedo, "Systematics of Moduli Stabilisation in Calabi-Yau Flux Compactifications," JHEP **0503**, 007 (2005) [arXiv:hep-th/0502058].

[28] K. Becker, M. Becker, M. Haack and J. Louis, "Supersymmetry breaking and alpha-prime corrections to flux induced potentials," JHEP **0206**, 060 (2002) [hep-th/0204254].

[29] R. Bousso and J. Polchinski, "Quantization of four form fluxes and dynamical neutralization of the cosmological constant," JHEP **0006**, 006 (2000) [hep-th/0004134].

[30] S. Ashok and M. R. Douglas, "Counting flux vacua," JHEP **0401**, 060 (2004) [hep-th/0307049].

[31] M. R. Douglas, "Basic results in vacuum statistics," Comptes Rendus Physique **5**, 965 (2004) [hep-th/0409207].

[32] F. Denef and M. R. Douglas, "Distributions of flux vacua," JHEP **0405**, 072 (2004) [hep-th/0404116].

[33] F. Denef and M. R. Douglas, "Distributions of nonsupersymmetric flux vacua," JHEP **0503**, 061 (2005) [hep-th/0411183].

[34] CMS Collaboration, CMS-PAS-SUS-11-016, "Interpretation of Searches for Supersymmetry," 2012.

[35] P. Draper, P. Meade, M. Reece and D. Shih, "Implications of a 125 GeV Higgs for the MSSM and Low-Scale SUSY Breaking," arXiv:1112.3068 [hep-ph].

[36] A. D. Sakharov, JETP Lett. **6**, 24 (1967).

[37] M. Dine and A. Kusenko, "The Origin of the Matter-Antimatter Asymmetry," Rev. Mod. Phys. **76** (2004) 1 [arXiv:hep-ph/0303065].

[38] S. Weinberg, "The Cosmological Constant Problem," Rev. Mod. Phys. **61**, 1 (1989).

[39] D. S. Dean and S. N. Majumdar, "Large Deviations of Extreme Eigenvalues of Random Matrices," Phys.Rev.Lett. **97** (2006) 160201, arXiv:cond-mat/0609651v2 [cond-mat.stat-mech].

[40] D. S. Dean and S. N. Majumdar, "Extreme Value Statistics of Eigenvalues of Gaussian Random Matrices," Phys. Rev. E **77**, 041108 (2008), arXiv:0801.1730v1 [cond-mat.stat-mech].

[41] D. Marsh, L. McAllister, T.Wrase, *forthcoming*.

[42] T. Bachlechner, D. Marsh, L. McAllister, T.Wrase, *forthcoming*.

[43] L. Randall and R. Sundrum, "Out of this world supersymmetry breaking," Nucl. Phys. B **557**, 79 (1999) [arXiv:hep-th/9810155].

[44] A. Anisimov, M. Dine, M. Graesser and S. D. Thomas, "Brane world SUSY breaking from string/M theory," JHEP **0203**, 036 (2002) [arXiv:hep-th/0201256].

[45] S. Kachru, J. McGreevy and P. Svrček, "Bounds on masses of bulk fields in string compactifications," JHEP **0604**, 023 (2006) [arXiv:hep-th/0601111].

[46] S. Kachru, L. McAllister and R. Sundrum, "Sequestering in string theory," JHEP **0710**, 013 (2007) [arXiv:hep-th/0703105].

[47] M. A. Luty and R. Sundrum, "Supersymmetry breaking and composite extra dimensions," Phys. Rev. D **65**, 066004 (2002) [arXiv:hep-th/0105137].

[48] A. E. Nelson and M. J. Strassler, "Exact results for supersymmetric renormalization and the supersymmetric flavor problem," JHEP **0207**, 021 (2002) [arXiv:hep-ph/0104051].

[49] K. Choi, K. S. Jeong and K. Okumura, "Phenomenology of mixed modulus-anomaly mediation in fluxed string compactifications and brane models," JHEP **0509**, 039 (2005) [arXiv:hep-ph/0504037].

[50] K. Inoue, M. Kawasaki, M. Yamaguchi and T. Yanagida, "Vanishing Squark and Slepton Masses in a Class of Supergravity Models," Phys. Rev. D **45** (1992) 328.

[51] M. A. Luty and R. Sundrum, "Radius stabilization and anomaly-mediated supersymmetry breaking," Phys. Rev. D **62**, 035008 (2000) [arXiv:hep-th/9910202].

[52] I. R. Klebanov and E. Witten, "Superconformal field theory on threebranes at a Calabi-Yau singularity," Nucl. Phys. B **536**, 199 (1998) [arXiv:hep-th/9807080].

[53] D. Baumann, A. Dymarsky, S. Kachru, I. R. Klebanov and L. McAllister, "Holographic Systematics of D-brane Inflation," JHEP **0903**, 093 (2009) [arXiv:0808.2811 [hep-th]].

[54] S. Kachru, J. Pearson and H. L. Verlinde, "Brane/Flux Annihilation and the String Dual of a Non-Supersymmetric Field Theory," JHEP **0206**, 021 (2002) [arXiv:hep-th/0112197].

[55] I. Bena, M. Graña and N. Halmagyi, "On the Existence of Meta-stable Vacua in Klebanov-Strassler," JHEP **1009**, 087 (2010) [arXiv:0912.3519 [hep-th]].

[56] A. Ceresole, G. Dall'Agata, R. D'Auria and S. Ferrara, "Spectrum of type IIB supergravity on AdS(5) x T**11: Predictions on N=1 SCFT's," Phys. Rev. **D61**, 066001 (2000) [hep-th/9905226].

[57] S. Kachru, R. Kallosh, A. D. Linde, J. Maldacena, L. McAllister and S. P. Trivedi, "Towards inflation in string theory," JCAP **0310**, 013 (2003) [hep-th/0308055].

[58] O. DeWolfe, S. Kachru and M. Mulligan, “A Gravity Dual of Metastable Dynamical Supersymmetry Breaking,” Phys. Rev. **D77**, 065011 (2008) [arXiv:0801.1520 [hep-th]].

[59] I. Bena, G. Giecold, M. Graña and N. Halmagyi, “On The Inflaton Potential From Antibranes in Warped Throats,” arXiv:1011.2626 [hep-th].

[60] D. Baumann, A. Dymarsky, S. Kachru, I. R. Klebanov and L. McAllister, “D3-brane Potentials from Fluxes in AdS/CFT,” JHEP **1006**, 072 (2010) [arXiv:1001.5028 [hep-th]].

[61] R. Blumenhagen, J. P. Conlon, S. Krippendorf, S. Moster and F. Quevedo, “SUSY Breaking in Local String/F-Theory Models,” JHEP **0909**, 007 (2009) [arXiv:0906.3297 [hep-th]].

[62] V. S. Kaplunovsky and J. Louis, “Model independent analysis of soft terms in effective supergravity and in string theory,” Phys. Lett. B **306**, 269 (1993) [arXiv:hep-th/9303040].

[63] A. Brignole, L. E. Ibáñez and C. Muñoz, “Soft supersymmetry-breaking terms from supergravity and superstring models,” arXiv:hep-ph/9707209.

[64] S. K. Soni and H. A. Weldon, “Analysis of the Supersymmetry Breaking Induced by $\mathcal{N} = 1$ Supergravity Theories,” Phys. Lett. B **126**, 215 (1983).

[65] J. P. Conlon, D. Cremades and F. Quevedo, “Kaehler potentials of chiral matter fields for Calabi-Yau string compactifications,” JHEP **0701**, 022 (2007) [arXiv:hep-th/0609180].

[66] K. Bobkov, V. Braun, P. Kumar and S. Raby, “Stabilizing All Kahler Moduli in Type IIB Orientifolds,” arXiv:1003.1982 [hep-th].

[67] M. Berg, M. Haack and B. Körs, “Loop corrections to volume moduli and inflation in string theory,” Phys. Rev. D **71**, 026005 (2005) [arXiv:hep-th/0404087].

[68] D. Baumann, A. Dymarsky, I. R. Klebanov, J. M. Maldacena, L. P. McAllister and A. Murugan, “On D3-brane potentials in compactifications with fluxes and wrapped D-branes,” JHEP **0611**, 031 (2006) [arXiv:hep-th/0607050].

[69] D. Baumann and L. McAllister, “A Microscopic Limit on Gravitational

Waves from D-brane Inflation," Phys. Rev. **D75**, 123508 (2007) [hep-th/0610285].

[70] I. R. Klebanov and M. J. Strassler, "Supergravity and a confining gauge theory: Duality cascades and chiSB-resolution of naked singularities," JHEP **0008**, 052 (2000) [arXiv:hep-th/0007191].

[71] O. DeWolfe, L. McAllister, G. Shiu and B. Underwood, "D3-brane Vacua in Stabilized Compactifications," JHEP **0709**, 121 (2007) [arXiv:hep-th/0703088].

[72] J. P. Conlon, "Mirror Mediation," JHEP **0803** (2008) 025 [arXiv:0710.0873 [hep-th]].

[73] K. Choi, K. S. Jeong and K. Okumura, "Flavor and CP conserving moduli mediated SUSY breaking in flux compactification," JHEP **0807**, 047 (2008) [arXiv:0804.4283 [hep-ph]].

[74] D. Baumann, A. Dymarsky, I. R. Klebanov and L. McAllister, "Towards an Explicit Model of D-brane Inflation," JCAP **0801**, 024 (2008) [arXiv:0706.0360 [hep-th]].

[75] S. Kuperstein, "Meson spectroscopy from holomorphic probes on the warped deformed conifold," JHEP **0503**, 014 (2005) [arXiv:hep-th/0411097].

[76] M. Berg, M. Haack and B. Körs, "On the moduli dependence of nonperturbative superpotentials in brane inflation," arXiv:hep-th/0409282.

[77] L. Girardello and M. T. Grisaru, "Soft Breaking of Supersymmetry," Nucl. Phys. B **194**, 65 (1982).

[78] K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, "Soft supersymmetry breaking in KKLT flux compactification," Nucl. Phys. B **718**, 113 (2005) [arXiv:hep-th/0503216].

[79] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, "A complete analysis of FCNC and CP constraints in general SUSY extensions of the standard model," Nucl. Phys. B **477**, 321 (1996) [arXiv:hep-ph/9604387].

[80] R. Blumenhagen, S. Moster and E. Plauschinn, "Moduli Stabilisation Ver-

sus Chirality for MSSM Like Type IIB Orientifolds," JHEP **0801**, 058 (2008) [arXiv:0711.3389 [hep-th]].

[81] S. P. de Alwis, "Classical and Quantum SUSY Breaking Effects in IIB Local Models," JHEP **1003** (2010) 078 [arXiv:0912.2950 [hep-th]].

[82] K. Choi, H. P. Nilles, C. S. Shin and M. Trapletti, "Sparticle Spectrum of Large Volume Compactification," arXiv:1011.0999 [hep-th].

[83] J. P. Conlon, F. Quevedo and K. Suruliz, "Large-Volume Flux Compactifications: Moduli Spectrum and D3/D7 Soft Supersymmetry Breaking," JHEP **0508**, 007 (2005) [arXiv:hep-th/0505076].

[84] M. Cvetič, J. Halverson and R. Richter, "Realistic Yukawa structures from orientifold compactifications," JHEP **0912**, 063 (2009) [arXiv:0905.3379 [hep-th]].

[85] M. Buican and S. Franco, "SUSY breaking mediation by D-brane instantons," JHEP **0812**, 030 (2008) [arXiv:0806.1964 [hep-th]].

[86] F. Marchesano and L. Martucci, "Non-perturbative effects on seven-brane Yukawa couplings," Phys. Rev. Lett. **104**, 231601 (2010) [arXiv:0910.5496 [hep-th]].

[87] J. P. Conlon, M. Goodsell and E. Palti, "Anomaly Mediation in Superstring Theory," arXiv:1008.4361 [hep-th].

[88] M. Haack, D. Krefl, D. Lüst, A. Van Proeyen and M. Zagermann, "Gaugino condensates and D-terms from D7-branes," JHEP **0701**, 078 (2007) [arXiv:hep-th/0609211].

[89] E. Dudas, Y. Mambrini, S. Pokorski and A. Romagnoni, "Moduli stabilization with Fayet-Iliopoulos uplift," JHEP **0804** (2008) 015 [arXiv:0711.4934 [hep-th]].

[90] P. McGuirk, G. Shiu and Y. Sumitomo, "Non-supersymmetric infrared perturbations to the warped deformed conifold," Nucl. Phys. B **842**, 383 (2010) [arXiv:0910.4581 [hep-th]].

[91] P. Candelas and X. C. de la Ossa, "Comments on Conifolds," Nucl. Phys. **B342**, 246 (1990).

[92] P. Ouyang, “Holomorphic D7 branes and flavored N=1 gauge theories,” Nucl. Phys. **B699**, 207 (2004) [hep-th/0311084].

[93] I. Affleck and M. Dine, “A New Mechanism for Baryogenesis,” Nucl. Phys. B **249** (1985) 361.

[94] K. Enqvist and A. Mazumdar, “Cosmological Consequences of MSSM Flat Directions,” Phys. Rept. **380** (2003) 99 [arXiv:hep-ph/0209244].

[95] A. Kusenko and M. E. Shaposhnikov, “Supersymmetric Q-Balls as Dark Matter,” Phys. Lett. B **418** (1998) 46 [arXiv:hep-ph/9709492].

[96] K. Enqvist and J. McDonald, “B-Ball Baryogenesis and the Baryon to Dark Matter Ratio,” Nucl. Phys. B **538** (1999) 321 [arXiv:hep-ph/9803380].

M. Fujii and K. Hamaguchi, “Higgsino and Wino Dark Matter from Q-Ball Decay in Affleck-Dine Baryogenesis,” Phys. Lett. B **525** (2002) 143 [arXiv:hep-ph/0110072].

M. Fujii and K. Hamaguchi, “Non-Thermal Dark Matter via Affleck-Dine Baryogenesis and Its Detection Possibility,” Phys. Rev. D **66** (2002) 083501 [arXiv:hep-ph/0205044].

M. Fujii and T. Yanagida, “A Solution to the Coincidence Puzzle of OmegA_B and Omega(Dm),” Phys. Lett. B **542** (2002) 80 [arXiv:hep-ph/0206066].

F. Doddato and J. McDonald, “Affleck-Dine Baryogenesis, Condensate Fragmentation and Gravitino Dark Matter in Gauge-Mediation with a Large Messenger Mass,” JCAP **1106** (2011) 008 [arXiv:1101.5328 [hep-ph]].

N. F. Bell, K. Petraki, I. M. Shoemaker and R. R. Volkas, “Pangenesis in a Baryon-Symmetric Universe: Dark and Visible Matter via the Affleck-Dine Mechanism,” arXiv:1105.3730 [hep-ph].

C. Cheung and K. M. Zurek, “Affleck-Dine Cogenesis,” arXiv:1105.4612 [hep-ph].

G. Kane, J. Shao, S. Watson and H. B. Yu, “The Baryon-Dark Matter Ratio via Moduli Decay After Affleck-Dine Baryogenesis,” JCAP **1111** (2011) 012 [arXiv:1108.5178 [hep-ph]].

[97] M. Dine, L. Randall and S. D. Thomas, “Baryogenesis from Flat Directions of the Supersymmetric Standard Model,” Nucl. Phys. B **458** (1996) 291 [arXiv:hep-ph/9507453].

M. Dine, L. Randall and S. D. Thomas, “Supersymmetry Breaking in the Early Universe,” Phys. Rev. Lett. **75** (1995) 398 [arXiv:hep-ph/9503303].

[98] C. F. Kolda and J. March-Russell, "Supersymmetric D-Term Inflation, Reheating and Affleck-Dine Baryogenesis," Phys. Rev. D **60** (1999) 023504 [arXiv:hep-ph/9802358].

A. Anisimov and M. Dine, "Some Issues in Flat Direction Baryogenesis," Nucl. Phys. B **619** (2001) 729 [arXiv:hep-ph/0008058].

R. Allahverdi, B. A. Campbell and J. R. Ellis, "Reheating and Supersymmetric Flat-Direction Baryogenesis," Nucl. Phys. B **579** (2000) 355 [arXiv:hep-ph/0001122].

R. Allahverdi and A. Mazumdar, "Reheating in Supersymmetric High Scale Inflation," Phys. Rev. D **76** (2007) 103526 [arXiv:hep-ph/0603244].

R. Allahverdi and A. Mazumdar, "Longevity of Supersymmetric Flat Directions," JCAP **0708** (2007) 023 [arXiv:hep-ph/0608296].

R. Allahverdi and A. Mazumdar, "Affleck-Dine Condensate, Late Thermalization and the Gravitino Problem," Phys. Rev. D **78** (2008) 043511 [arXiv:0802.4430 [hep-ph]].

[99] B. Dutta and K. Sinha, "Affleck-Dine Baryogenesis in Effective Supergravity," Phys. Rev. D **82** (2010) 095003 [arXiv:1008.0148 [hep-th]].

[100] J. A. Casas and G. B. Gelmini, "Conditions for Viable Affleck-Dine Baryogenesis: Implications for String Theories," Phys. Lett. B **410** (1997) 36 [arXiv:hep-ph/9706439].

[101] S. Kasuya and M. Kawasaki, "Towards the Robustness of the Affleck-Dine Baryogenesis," Phys. Rev. D **74** (2006) 063507 [arXiv:hep-ph/0606123].

[102] V. S. Kaplunovsky and J. Louis, "Model Independent Analysis of Soft Terms in Effective Supergravity and in String Theory," Phys. Lett. B **306** (1993) 269 [arXiv:hep-th/9303040].

[103] T. Gherghetta, C. F. Kolda and S. P. Martin, "Flat Directions in the Scalar Potential of the Supersymmetric Standard Model," Nucl. Phys. B **468** (1996) 37 [arXiv:hep-ph/9510370].

[104] A. Basboll, "A Complete and Minimal Catalogue of MSSM Gauge Invariant Monomials," Int. J. Mod. Phys. A **25** (2010) 3375 [arXiv:0910.0244 [hep-ph]].

[105] M. Senami and K. Yamamoto, "Flat Manifold Leptogenesis in the Supersymmetric Standard Model," Phys. Rev. D **66** (2002) 035006 [arXiv:hep-ph/0205041].

M. Senami and T. Takayama, "Affleck-Dine Leptogenesis via Multiscalar Evolution in a Supersymmetric Seesaw Model," JCAP **0711** (2007) 015 [arXiv:0708.2238 [hep-ph]].
K. Kamada and J. Yokoyama, "Affleck Dine Leptogenesis via Multiple Flat Directions," Phys. Rev. D **78** (2008) 043502 [arXiv:0803.3146 [hep-ph]].

[106] K. Enqvist, A. Jokinen and A. Mazumdar, "Dynamics of MSSM Flat Directions Consisting of Multiple Scalar Fields," JCAP **0401** (2004) 008 [arXiv:hep-ph/0311336].

[107] E. Dudas and S. K. Vempati, "General Soft Terms from Supergravity Including D-Terms," arXiv:hep-ph/0506029.

[108] S. I. Goldberg and S. Kobayaski, "Holomorphic Bisectonal Curvature," J. Differential Geometry **1** (1967) 225.

[109] H. Seshadri and F. Zheng, "Complex Product Manifolds Cannot be Negatively Curved," Asian J. Math. **12** (2008) 145.

[110] J. P. Conlon, R. Kallosh, A. D. Linde and F. Quevedo, "Volume Modulus Inflation and the Gravitino Mass Problem," JCAP **0809** (2008) 011 [arXiv:0806.0809 [hep-th]].

[111] L. Randall and R. Sundrum, "Out of This World Supersymmetry Breaking," Nucl. Phys. B **557** (1999) 79 [arXiv:hep-th/9810155].

[112] S. Kachru, L. McAllister and R. Sundrum, "Sequestering in String Theory," JHEP **0710** (2007) 013 [arXiv:hep-th/0703105].

[113] R. Kallosh and A. D. Linde, "Landscape, the Scale of SUSY Breaking, and Inflation," JHEP **0412** (2004) 004 [arXiv:hep-th/0411011].

[114] M. K. Gaillard, H. Murayama and K. A. Olive, "Preserving Flat Directions during Inflation," Phys. Lett. B **355** (1995) 71 [arXiv:hep-ph/9504307].

[115] J. A. Adams, G. G. Ross and S. Sarkar, "Multiple Inflation," Nucl. Phys. B **503** (1997) 405 [arXiv:hep-ph/9704286].

[116] C. Gordon, D. Wands, B. A. Bassett and R. Maartens, "Adiabatic and Entropy Perturbations from Inflation," Phys. Rev. D **63** (2001) 023506 [arXiv:astro-ph/0009131].

[117] C. M. Peterson and M. Tegmark, "Testing Two-Field Inflation," Phys. Rev. D **83** (2011) 023522 [arXiv:1005.4056 [astro-ph.CO]].

[118] L. Senatore and M. Zaldarriaga, "The Effective Field Theory of Multifield Inflation," arXiv:1009.2093 [hep-th].

[119] D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, "Primordial Features Due to a Step in the Inflaton Potential," JCAP **1010** (2010) 008 [arXiv:1005.2175 [astro-ph.CO]].

[120] J. A. Adams, B. Cresswell and R. Easther, "Inflationary Perturbations from a Potential with a Step," Phys. Rev. D **64** (2001) 123514 [arXiv:astro-ph/0102236].

[121] H. V. Peiris *et al.* [WMAP Collaboration], "First Year Wilkinson Microwave Anisotropy Probe (Wmap) Observations: Implications for Inflation," Astrophys. J. Suppl. **148** (2003) 213 [arXiv:astro-ph/0302225].

[122] L. Covi, J. Hamann, A. Melchiorri, A. Slosar and I. Sorbera, "Inflation and WMAP Three Year Data: Features Have a Future!," Phys. Rev. D **74** (2006) 083509 [arXiv:astro-ph/0606452].

J. Hamann, L. Covi, A. Melchiorri and A. Slosar, "New Constraints on Oscillations in the Primordial Spectrum of Inflationary Perturbations," Phys. Rev. D **76** (2007) 023503 [arXiv:astro-ph/0701380].

[123] M. J. Mortonson, C. Dvorkin, H. V. Peiris and W. Hu, "Cmb Polarization Features from Inflation Versus Reionization," Phys. Rev. D **79** (2009) 103519 [arXiv:0903.4920 [astro-ph.CO]].

[124] P. Hunt and S. Sarkar, "Multiple Inflation and the WMAP 'Glitches'," Phys. Rev. D **70** (2004) 103518 [arXiv:astro-ph/0408138].

[125] P. Hunt and S. Sarkar, "Multiple Inflation and the WMAP 'Glitches' II. Data Analysis and Cosmological Parameter Extraction," Phys. Rev. D **76** (2007) 123504 [arXiv:0706.2443 [astro-ph]].

[126] D. Wands, N. Bartolo, S. Matarrese and A. Riotto, "An Observational Test of Two-Field Inflation," Phys. Rev. D **66** (2002) 043520 [arXiv:astro-ph/0205253].

[127] A. A. Abolhasani, H. Firouzjahi and M. H. Namjoo, "Curvature Perturba-

tions and Non-Gaussianities from Waterfall Phase Transition during Inflation," Class. Quant. Grav. **28** (2011) 075009 [arXiv:1010.6292 [astro-ph.CO]].

[128] M. Joy, A. Shafieloo, V. Sahni and A. A. Starobinsky, "Is a Step in the Primordial Spectral Index Favored by Cmb Data ?," JCAP **0906** (2009) 028 [arXiv:0807.3334 [astro-ph]].

M. Joy, V. Sahni and A. A. Starobinsky, "A New Universal Local Feature in the Inflationary Perturbation Spectrum," Phys. Rev. D **77** (2008) 023514 [arXiv:0711.1585 [astro-ph]].

R. K. Jain, P. Chingangbam, J. O. Gong, L. Sriramkumar and T. Souradeep, "Double Inflation and the Low Cmb Multipoles," JCAP **0901** (2009) 009 [arXiv:0809.3915 [astro-ph]].

[129] K. Enqvist and J. McDonald, "Inflationary Affleck-Dine Scalar Dynamics and Isocurvature Perturbations," Phys. Rev. D **62** (2000) 043502 [arXiv:hep-ph/9912478].

[130] D. Larson *et al.*, "Seven-Year Wilkinson Microwave Anisotropy Probe (Wmap) Observations: Power Spectra and Wmap-Derived Parameters," Astrophys. J. Suppl. **192** (2011) 16 [arXiv:1001.4635 [astro-ph.CO]].

E. Komatsu *et al.* [WMAP Collaboration], "Seven-Year Wilkinson Microwave Anisotropy Probe (Wmap) Observations: Cosmological Interpretation," Astrophys. J. Suppl. **192** (2011) 18 [arXiv:1001.4538 [astro-ph.CO]].

[131] M. L. Brown *et al.* [QUaD collaboration], "Improved Measurements of the Temperature and Polarization of the Cmb from Quad," Astrophys. J. **705** (2009) 978 [arXiv:0906.1003 [astro-ph.CO]].

[132] C. L. Reichardt *et al.*, "High resolution CMB power spectrum from the complete ACBAR data set," Astrophys. J. **694**, 1200 (2009) [arXiv:0801.1491 [astro-ph]].

[133] S. Hotchkiss and S. Sarkar, "Non-Gaussianity from Violation of Slow-Roll in Multiple Inflation," JCAP **1005** (2010) 024 [arXiv:0910.3373 [astro-ph.CO]].

[134] J. Hamann, A. Shafieloo and T. Souradeep, "Features in the Primordial Power Spectrum? a Frequentist Analysis," JCAP **1004** (2010) 010 [arXiv:0912.2728 [astro-ph.CO]].

[135] X. Chen, R. Easther and E. A. Lim, "Large Non-Gaussianities in Single Field Inflation," JCAP **0706** (2007) 023 [arXiv:astro-ph/0611645].

[136] G. Shiu and J. Xu, "Effective Field Theory and Decoupling in Multi-Field Inflation: an Illustrative Case Study," arXiv:1108.0981 [hep-th].

[137] J. P. Conlon and F. Quevedo, "Kahler moduli inflation," JHEP **0601**, 146 (2006) [arXiv:hep-th/0509012].

[138] M. Cicoli and A. Mazumdar, "Reheating for Closed String Inflation," JCAP **1009** (2010) 025 [arXiv:1005.5076 [hep-th]].

[139] A. Vilenkin, "Quantum Fluctuations in the New Inflationary Universe," Nucl. Phys. B **226** (1983) 527.

[140] L. J. Hall and L. Randall, "Weak Scale Effective Supersymmetry," Phys. Rev. Lett. **65** (1990) 2939.

[141] S. Mukhi, "The Geometric Background Field Method, Renormalization and the Wess-Zumino Term in Nonlinear Sigma Models," Nucl. Phys. B **264** (1986) 640.

[142] M. R. Douglas and S. Kachru, "Flux compactification," Rev. Mod. Phys. **79**, 733-796 (2007) [hep-th/0610102].

[143] C. Tracy and H. Widom, "Level-spacing distributions and the Airy kernel," Commun. Math. Phys. **159**, 151-174, 1994.

[144] A. Aazami and R. Easther, "Cosmology from random multifield potentials," JCAP **0603**, 013 (2006) [hep-th/0512050].

[145] L. Covi, M. Gomez-Reino, C. Gross, J. Louis, G. A. Palma, and C. A. Scrucca, "de Sitter vacua in no-scale supergravities and Calabi-Yau string models," JHEP **0806**, 057 (2008) [arXiv:0804.1073 [hep-th]].

[146] D. V. Voiculescu, K. J. Dykema, and A. Nica, *Free random variables,* CRM Monograph Series **1**, American Mathematical Society, Providence, RI, 1992.

[147] F. Ferrari, S. Klevtsov, and S. Zelditch, "Random Kahler Metrics," arXiv:1107.4575 [hep-th].

[148] A. Edelman and N. R. Rao, "Random matrix theory," Acta Numerica, pp. 1-65, 2005.

[149] M. L. Mehta, *Random Matrices*, Academic Press, Boston, 1991.

[150] E. P. Wigner, “On the statistical distribution of the widths and spacings of nuclear resonance levels,” Math. Proc. Cambridge Philos. Soc. **47**, 790 (1951).

[151] E. P. Wigner, “Characteristic vectors of bordered matrices with infinite dimensions,” Annals of Mathematics **62**, 548-564, 1955.

[152] E. P. Wigner, “Results and theory of resonance absorbtion,” Oak Ridge Natl. Lab. Rept. ORNL-**2309**, 1957, 59–70;
“Statistical properties of real symmetric matrices with many dimensions,” Can. Math. Congr. Proc., University of Toronto Press, Toronto, Canada, 1957, 174–184.

[153] F. J. Dyson, “Statistical theory of the energy levels of complex systems, I,” J. Math. Phys. **3** 140 (1962); “A Brownian motion model for the eigenvalues of a random matrix,” J. Math. Phys. **3** 1191 (1962).

[154] J. Wishart, “The generalized product moment distribution in samples from a normal multivariate population,” Biometrika **20A**, 32 (1928).

[155] A. Edelman, “The Distribution and Moments of the Smallest Eigenvalue of a Random Matrix of Wishart Type,” Linear Algebra and Its Applications, **159**, 55, (1991).

[156] V. A. Marchenko and L. A. Pastur, “Distributions of eigenvalues of some sets of random matrices,” Math. USSR-Sb **1**, 507-536, 1967.

[157] A. Altland and M. R. Zirnbauer, “Nonstandard symmetry classes in mesoscopic normal-superconducting hybrid structures,” Phys. Rev. B **55**, 1142 - 1161 (1997).

[158] P. J. Forrester, “The spectrum edge of random matrix ensembles,” Nucl. Phys. B **402**, 709-728 (1993).

[159] C. Nadal, S. N. Majumdar, “A simple derivation of the Tracy-Widom distribution of the maximal eigenvalue of a Gaussian unitary random matrix”, J. Stat. Mech. (2011) P04001, arXiv:1102.0738v3 [cond-mat.stat-mech].

[160] I. M. Johnstone, “On the distribution of the largest eigenvalue in principal components analysis,” Ann. Statist. **29** (2001), no. 2, 295–327.

[161] K. Johansson, “Shape fluctuations and random matrices,” Comm. Math. Phys. **209**, 437-476, 2000.

[162] G. Ben Arous, A. Dembo, and A. Guionnet, “Aging of spherical spin glasses,” Probab. Theory Relat. Fields, **120**, 1-67 (2001).

[163] A. J. Bray and D. S. Dean, “The statistics of critical points of Gaussian fields on large-dimensional spaces,” arXiv:cond-mat/0611023v1 [cond-mat.dis-nn].

[164] P. Vivo, S. N. Majumdar, and O. Bohigas, “Large Deviations of the Maximum Eigenvalue in Wishart Random Matrices,” J. Phys. A: Math. Theor. **40** (16) (2007) 4317-4337, arXiv:cond-mat/0701371v2 [cond-mat.stat-mech].

[165] S. N. Majumdar and M. Vergassola, “Large Deviations of the Maximum Eigenvalue for Wishart and Gaussian Random Matrices,” Phys. Rev. Lett. **102**, 060601 (2009), arXiv:0811.2290.

[166] S. N. Majumdar, C. Nadal, A. Scardicchio, and P. Vivo, “The Index Distribution of Gaussian Random Matrices,” Phys. Rev. Lett. **103**, 220603 (2009), arXiv:0910.0775v1 [cond-mat.stat-mech].

[167] E. Katzav and I. P. Castillo, “Large Deviations of the Smallest Eigenvalue of the Wishart-Laguerre Ensemble,” arXiv:1005.5058v3 [cond-mat.dis-nn].

[168] R. Speicher, “Multiplicative functions on the lattice of non-crossing partitions and free convolution,” Math. Ann. **298** (1994), 611-628.

[169] A. Edelman and N. R. Rao, “The polynomial method for random matrices,” Foundations of Computational Mathematics, arXiv:math/0601389v3 [math.PR].

[170] M. Abramowitz and I. A. Stegun, “Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables,” Dover, New York, 1964.

[171] H. Solomon and M. A. Stephens, “Distribution of a Sum of Weighted Chi-Square Variables,” Journal of the American Statistical Association **72** 360 (1977), 881-885.

[172] K. A. Takeuchi and M. Sano, “Universal Fluctuations of Growing Interfaces: Evidence in Turbulent Liquid Crystals,” Phys. Rev. Lett. **104**, 230601 (2010).

[173] J. Baik, A. Borodin, P. Deift, and T. A. Suidan, "A model for the bus system in Cuernavaca (Mexico)," J. Phys. A **39** (28) (2006), 8965 - 8975; M. Krbálek and P. Šeba, "Statistical properties of the city transport in Cuernavaca (Mexico) and random matrix theory," J. Phys. A: Math. Gen. **33** (2000), 229 - 234.

[174] M. Fridman et. al., "Measuring maximal eigenvalue distribution of Wishart random matrices with coupled lasers," arXiv: 1012.1282.

[175] P. Deift, "Universality for mathematical and physical systems," arXiv:math-ph/0603038v2.

[176] A. B. J. Kuijlaars, "Universality," Chapter 6 in *Oxford Handbook of Random Matrix Theory*, G. Akemann, J. Baik, and P. Di Francesco, eds., Oxford University Press, 2011. arXiv:1103.5922v2 [math-ph].

[177] Z. D. Bai, "Methodologies in spectral analysis of large dimensional random matrices, a review," Statistica Sinica **9**, 611-677.

[178] A. Soshnikov, "A note on universality of the distribution of the largest eigenvalues in certain sample covariance matrices," J. Statist. Phys. **108**, 1033–1056, 2002.

[179] G. Biroli, J-P. Bouchaud, and M. Potters, "On the top eigenvalue of heavy-tailed random matrices," cond-mat/0609070.

[180] A. Borghese, R. Linares and D. Roest, "Minimal Stability in Maximal Supergravity," arXiv:1112.3939 [hep-th].

[181] J. H. Schenker and H. Schulz-Baldes, "Semicircle law and freeness for random matrices with symmetries or correlations," Math. Res. Lett. **12** (2005), 531-542, arXiv:math-ph/0505003v1.

[182] M. R. Douglas, B. Shiffman, and S. Zelditch, "Critical points and supersymmetric vacua, I, II, III," Commun. Math. Phys. **252** (2004) 325-358, arXiv:math/0402326v2 [math.CV]; J. Diff. Geometry **72** (2006), 381-427, arXiv:math/0406089v3 [math.CV]; Commun. Math. Phys. **265**:617-671, 2006, arXiv:math-ph/0506015v4.

[183] F. Denef, M. R. Douglas, and B. Florea, "Building a better racetrack," JHEP **0406**, 034 (2004) [hep-th/0404257].

[184] F. Denef, M. R. Douglas, B. Florea, A. Grassi, and S. Kachru, "Fixing all moduli in a simple F-theory compactification," Adv. Theor. Math. Phys. **9**, 861 (2005) [hep-th/0503124].

[185] X. Chen, G. Shiu, Y. Sumitomo and S. H. H. Tye, "A Global View on The Search for de-Sitter Vacua in (type IIA) String Theory," arXiv:1112.3338 [hep-th].